Econometric history of the growth-volatility relationship in the U.S.: 1919-2017*

Amélie CHARLES†

Audencia Business School

Olivier DARNÉ‡

LEMNA, University of Nantes

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†Audencia Business School, 8 route de la Joneli, 44312 Nantes Cedex 3. Email: acharles@audencia.com.

‡Corresponding author: LEMNA, University of Nantes, Chemin de la Censive du Tertre, BP 52231, 44322 Nantes, France. Email: olivier.darne@univ-nantes.fr.

Abstract

In this paper, we investigate the relationship between output volatility and growth using the standard GARCH-M framework and the U.S. monthly industrial production index (IPI) for the period January 1919 to December 2017, by taking into account the presence of shocks and variance changes. The results show that the IPI growth is strongly affected by large shocks which are associated with strikes in some industries, recessions, World War II and natural disasters. We also identify several subperiods with different level of volatility where the volatility declines along the subperiods, with the pre-WWII period (1919-1946) the highest volatile period and the aftermath period of the GFC (2010-2017) the lowest volatile period. We find no evidence of relationship between output volatility and its growth during the full sample 1919-2017 and also for all the subperiods. From a macroeconomic point of view, this implies that economic performances, as measured by IPI growth, do not depend on the uncertainty as measured by IPI volatility.

Keywords: Growth-volatility relationship; breaks; shock; GARCH-M model; cliometrics.

JEL Classification: E32; C22; N12; O40.

1 Introduction

The relationship between output volatility and growth is an important issue in the economics literature. For example, Schumpeter (1939) emphasized the idea of creative destruction predicting a positive relationship, while Arrow (1962) highlights the idea of learning-by-doing leading to a negative relationship. More recent theoretical work also covers the real option literature where firms view their investment choices as a series of options. As volatility increases the option value of delay, firms postpone their investment and so growth is negatively affected (Bernanke, 1983).

A number of studies examine the issue of how the volatility of the output growth rate affects the growth rate of output. That is, does decreased output growth rate volatility cause a higher or lower output growth rate? Alternative theoretical models give mixed results: Negative (Martin and Rogers, 1997, 2000), positive (e.g., Black, 1987; Blackburn, 1999), or independent relationships (Phelps 1968; Lucas 1972) between output growth volatility and output growth.[1] In the empirical literature the Generalized Autoregressive Heteroscedasticity in Mean (GARCH-M) model developed by Engle et al. (1987) is often used to investigate the volatility-growth relationship, taking into account the time varying nature of output volatility. As shown by Fernández-Villaverde and Rubio-Ramírez (2010), time-varying volatility, namely periods of high volatility followed by periods of low volatility, is an important feature of macroeconomic times series. Empirical evidence of the growth-volatility relationship, however, is also mixed: negative (e.g., Henry and Olekalns, 2002), positive (e.g., Fountas and Karanasos, 2006; Lee, 2010; Fang and Miller, 2014), or no statistically significant relationship (e.g., Grier and Perry, 2000; Fang et al., 2008).

However, previous studies ignore a number of issues that could be important in

[1] See Fang and Miller (2008) for a discussion of the growth-volatility relationship.

identifying the true relationship between output volatility and growth. First, existing studies using the GARCH-M framework do not include the possibility that volatility could be more affected by negative shocks than positive shocks. For example, Brunner (1997), Henry and Olekalns (2002) and Ho and Tsui (2003) find evidence of asymmetric volatility in the output growth rates in the U.S. This potential nonlinearity in the variance of growth should also be addressed. To incorporate the asymmetric effect of positive and negative shocks to the variance of growth, Trypsteen (2017) suggests that the variance equation could include a nonlinear term as in the GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993).

Second, Balke and Fomby (1991), Darné and Diebolt (2004) or Darné (2009), inter alia, show that specific events have a dramatic impact on modelling macroeconomic time series for the U.S. This type of event includes, for example, oil shocks, wars, financial slumps, changes of policy regimes, recessions, natural disasters, etc. Due to their unpredictable nature and large impact on macroeconomic relationships, these extraordinary events are referred to as (infrequent) large shocks and are often identified as breaks, jumps or outliers, that is exogenous changes that directly affect the series.
One way to identify (infrequent) large shocks is intervention analysis, introduced by Box and Tiao (1975) to attempt to statistically appraise these types of shocks (or outliers or jumps). Intervention analysis is used to assess the impact of a known or unknown event on the time series. The main focus is to estimate the effect of such events on the series. Intervention analysis forms the basis for many outlier modelling procedures. Moreover, it is well known that these shocks may pose difficulties for the identification and estimation of volatility models (see, e.g., Charles, 2008; Carnero et al., 2001, 2006; Charles and Darné; 2014).

Finally, some shocks can cause abrupt breaks in the unconditional variance and are equivalent to structural breaks in the parameters of the GARCH process governing the conditional volatility. Ignoring structural breaks in the time-varying variance of a

GARCH model can bias the GARCH parameters towards one (see, e.g., Lamoureux and Lastrapes, 1990; Hillebrand, 2005). For example, McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), Stock and Watson (2003) and Cecchetti et al. (2006), among others, document a structural change in the volatility of U.S. output growth, finding a rather dramatic reduction in output volatility, known as the Great Moderation. Using GARCH specifications with breaks in volatility, Fang et al. (2008) and Fang and Miller (2008) show that the time-varying variance falls sharply or disappears, once they incorporate the break in the variance equation of output.

Further, most of existing studies analyze the volatility-growth relationship during the post-WWII period, except the studies of Fountas and Karanasos (2006) and Fang and Miller (2014) over the periods the 1860-1999 and 1876-2012, respectively. Moreover, when examining the relationship the output growth is often defined by the GDP/GNP growth in most of previous studies, from ever annual data (e.g., Ramey and Ramey, 1995; Caporale and McKierman, 1998; Fountas and Karanasos, 2006) or quarterly data (e.g., Fang et al., 2008; Fang and Miller, 2014; Charles et al., 2018), namely low frequency data. To the best of our knowledge, only Grier and Perry (2000) and Trypsteen (2017) examine the volatility-growth relationship using industrial production index as output at a monthly frequency, namely high frequency (in macroeconomics), which seems better to fit GARCH models, but only on the post-WWII period. Further, these both studies do not take into account the presence of outliers in the volatility.

This paper investigates the relationship between output volatility and growth using the standard GARCH-M framework and a long series of the U.S. monthly industrial production index for the period January 1919 to December 2017. This historical period allows to compare the pre- and post-World War II. This study analyzes the growth-volatility relationship accordingly to account for the issues discussed above. Firstly, we employ the semi-parametric procedure to detect jumps proposed by Laurent, Lecourt and Palm (LLP) (2016) based on the GARCH and GJR-GARCH models, and try to associate these shocks to political, financial or economic events. Secondly, we use an appropriate methodology to identify breakpoints and sudden shifts in volatility to define low volatility periods and high volatility periods. A relatively recent approach to test for volatility shifts is the iterative cumulative sums of squares (ICSS) algorithm (Inclán and Tiao, 1994; Sansó et al., 2004). The variance changes detected allow to define sub-periods for which we examine the evolution of the growth-volatility relationship using both GARCH-in-Mean and GJR-GARCH-in-Mean models.

The remainder of this paper is organized as follows: Section 2 describes the methodology of jump and variance-change detection, and presents the results. The GARCH and GJR-GARCH models as well as the effects of jumps and variance changes on output volatility modeling are presented in Section 3. Section 4 discussed the growth-volatility relationship for the full period and the subperiods. Finally, Section 5 concludes.

2 Detection of outliers and variance changes

2.1 Jump detection in GARCH models

We apply the semi-parametric procedure to detect jumps proposed by Laurent et al. (LLP) (2016). Their test is similar to the non-parametric tests for jumps in Lee and Mykland (2008) and Andersen, Bollerslev, and Dobrev (2007) for monthly data.

LLP assume that the growth rate series y_t is described by a Normal AR(p)-

GARCH(1,1) model

$$r_t = \mu_t + \varepsilon_t \tag{1}$$

$$\phi(L)\mu_t = c + \varepsilon_t \qquad \Leftrightarrow \qquad \mu_t = c + \sum_{i=1}^{\infty} \xi_i \varepsilon_{t-i} \tag{2}$$

$$\varepsilon_t = \sigma_t z_t, \qquad z_t \sim i.i.d.N(0,1), \tag{3}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{4}$$

where ξ_i are the coefficients of $\xi(L) = \phi^{-1}(L) = 1 + \sum_{i=1}^{\infty} \xi_i L^i$, L is the lag operator, and $\phi(L)$ is the AR polynomial of order p . The parameters should satisfy $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ to guarantee the positivity of the conditional variance.

Consider the growth rate series y_t with an independent jump component $a_t I_t$, defined as

$$y_t^* = y_t + a_t I_t \tag{5}$$

where y_t^* denotes the observed series, I_t is a dummy variable for a jump on month t, and a_t is the jump size. In equation (5) a jump $a_t I_t$ will not affect σ_{t+1}^2 (the conditional variance of r_{t+1}), so that we can have non-Gaussian fat-tailed conditional distributions of y_t^*.

LLP then use the bounded innovation propagation (BIP)-AR proposed by Muler, Peña and Yohai (2009) and the BIP-GARCH(1,1) of Muler and Yohai (2008) to obtain robust estimates of μ_t and σ_t^2 respectively in equations (1) and (4). These are denoted by $\widetilde{\mu}_t$ and $\widetilde{\sigma}_t$ and are robust to potential jumps $a_t I_t$ (i.e. they are estimated on y_t^* and not on y_t). The BIP-AR and BIP-GARCH(1,1) are defined as

$$\widetilde{\mu}_t = \mu + \sum_{i=1}^{\infty} \xi_i \widetilde{\sigma}_{t-i} \omega_{k_\delta}^{MPY}(\widetilde{J}_{t-i}) \tag{6}$$

$$\widetilde{\sigma}_t^2 = \omega + \alpha_1 \widetilde{\sigma}_{t-1}^2 c_\delta \omega_{k_\delta}^{MPY}\left(\widetilde{J}_{t-1}\right)^2 + \beta_1 \widetilde{\sigma}_{t-1}^2 \tag{7}$$

where ξ_i are the coefficients from the AR(p) polynomial defined in equation (2), $\omega_{k_\delta}^{MPY}(.)$ is the weight function, and c_δ a factor ensuring that the conditional expectation of the weighted square of unexpected shocks equals the conditional variance of r_t in the absence of jumps (Boudt et al., 2013).

Consider the standardized growth-rate series on month t given by

$$\widetilde{J}_t = \frac{y_t^* - \widetilde{\mu}_t}{\widetilde{\sigma}_t} \tag{8}$$

LLP detect the presence of jumps by testing the null hypothesis $\mathrm{H}_0 : a_t I_t = 0$ against the alternative $\mathrm{H}_1 : a_t I_t \neq 0$. The null is rejected if

$$\max_T |\widetilde{J}_t| > g_{T,\lambda}, \qquad t = 1, \ldots, T \tag{9}$$

where $g_{T,\lambda}$ is the suitable critical value.[2] If H_0 is rejected a dummy variable is defined as follows

$$\widetilde{I}_t = I\left(|\widetilde{J}_t| > k\right) \tag{10}$$

where $I(.)$ is the indicator function, with $\widetilde{I}_t = 1$ when a jump is detected at time t. LLP show that their test does not suffer from size distortions irrespective of the parameter values of the GARCH model in Monte Carlo simulations. The adjusted returns $\widetilde{y}_t$ are obtained as follows

$$\widetilde{y}_t = y_t^* - (y_t^* - \widetilde{\mu}_t)\widetilde{I}_t \tag{11}$$

Laurent et al. (2016) extend this test for additive jumps in AR-GARCH models, called BIP-AR-BIP-GARCH models, to AR-GJR-GARCH models, called BIP-AR-BIP-GJR models, to account for asymmetric effects.[3]

[2]The critical values are defined by $g_{T,\lambda} = -\log(-\log(1-\lambda))\, b_T + c_T$, with $b_T = 1/\sqrt{2\log T}$, and $c_T = (2\log T)^{1/2} - [\log\pi + \log(\log T)]/[2(2\log T)^{1/2}]$. Laurent et al. (2016) suggest setting $\lambda = 0.5$.

[3]Fang et al. (2014) also detect and correct the outliers in the growth rate of real GNP using another approach. They first detect the outliers if $|y_t - \bar{y}| > k \times \sigma_y$, where k measures the stringency imposed on outlier detection, and then apply the method of Ané et al. (2008) to correct the outliers identified. However, this approach is very sensitive to the value of k since when $k = 2$, 3 and 4 it identifies 36, 7 and 1 outliers, respectively. The choice of $k = 3$ by Fang et al. (2014) is only based on the number of outliers found in previous studies.

2.2 Variance change detection

The most popular statistical methods specifically designed to detect breaks in volatility are CUSUM-type tests. Here, we use the modified iterative cumulative sum of squares (ICSS) algorithm proposed by Sansó et al. (2004) which is based on the ICSS algorithm developed by Inclán and Tiao (1994).

Let i_t the IPI growth rate. $\{y_t\}$ is then assumed to be a series of independent observations from a Normal distribution with zero mean and unconditional variance σ_t^2 for $t = 1,\ldots,T$. Assume that the variance within each interval is denoted by σ_j^2, $j = 0,1,\ldots,N_T$, where N_T is the total number of variance changes and $1 < \kappa_1 < \kappa_2 < \cdots < \kappa_{N_T} < T$ are the set of breakpoints. Then the variances over the N_T intervals are defined as

$$\sigma_t^2 = \begin{cases} \sigma_0^2, & 1 < t < \kappa_1 \\ \sigma_1^2, & \kappa_1 < t < \kappa_2 \\ \cdots & \\ \sigma_{N_T}^2, & \kappa_{N_T} < t < T \end{cases}$$

The cumulative sum of squares is used to estimate the number of variance changes and to detect the point in time of each variance shift. The cumulative sum of the squared observations from the beginning of the series to the kth point in time is expressed as $C_k = \sum_{t=1}^{k} y_t^2$ for $k = 1,\ldots,T$. In order to test the null hypothesis of constant unconditional variance, the Inclán-Tiao statistic is given by:

$$IT = \sup_k |(T/2)^{0.5} D_k| \qquad (12)$$

where $D_k = \left(\frac{C_k}{C_T}\right) - \left(\frac{k}{T}\right)$, with C_T is the sum of the squared residuals from the whole sample period. The value of k that maximizes $|(T/2)^{0.5} D_k|$ is the estimate of the break date. The ICSS algorithm systematically looks for breakpoints along the sample. If there are no variance shifts over the whole sample period, D_k will oscillate around zero. Otherwise, if there are one or more variance shifts, D_k will departure from zero. The asymptotic distribution of IT is given by $sup_r|W^*(r)|$, where $W^*(r) = W(r) - rW(1)$ is a Brownian bridge and $W(r)$ is standard Brownian motion. Finite-sample critical values can be generated by simulation.

The IT statistic is designed for i.i.d. processes, which can be a strong assumption for macroeconomic data, especially for high frequency data, in which there is evidence of conditional heteroskedasticity. Sansó et al. (2004) show that the size distortions are important for heteroskedastic conditional variance processes from Monte carlo simulations. To overcome this problem, Sansó et al. (2004) propose a new test that explicitly consider the fourth moment properties of the disturbances and the conditional heteroskedasticity. They suggest a non-parametric adjustment to the IT statistic that allows y_t to obey a wide class of dependent processes under the null hypothesis. As suggested by Sansó et al. (2004), we use a non-parametric adjustment based on the Bartlett kernel, and the adjusted statistic is given by:

$$AIT = \sup_k |T^{-0.5} G_k| \tag{13}$$

where $G_k = \hat{\lambda}^{-0.5}\left[C_k - \left(\frac{k}{T}\right)C_T\right]$, $\hat{\lambda} = \hat{\gamma}_0 + 2\sum_{l=1}^{m}\left[1 - l(m+1)^{-1}\right]\hat{\gamma}_l$, $\hat{\gamma}_l = T^{-1}\sum_{t=l+1}^{T}(e_t^2 - \hat{\sigma}^2)(e_{t-l}^2 - \hat{\sigma}^2)$, $\hat{\sigma}^2 = T^{-1}C_T$, and the lag truncation parameter m is selected using the procedure in Newey and West (1994). Under general conditions, the asymptotic distribution of AIT is also given by $sup_r|W^*(r)|$, and finite-sample critical values can be generated by simulation.

Rodrigues and Rubia (2011) discuss the effects that sample contamination has on the asymptotic properties of CUSUM-type tests for detecting change points in variance and characterize the finite sample behavior by means of Monte Carlo simulations.

They focus on additive outliers which prove able to generate large size distortions in these tests. The authors show that the Sansó et al. (2004) test exhibits low power and tends to find few or no breaks at all. Therefore, as suggested by Rodrigues and Rubia (2011), we attempt to identify the variance changes not from the original data but from the outlier-adjusted data.[4]

2.3 Results

We study the U.S. monthly industrial production index (IPI) for the period January 1919 to December 2017, namely 1,188 observations. The data are downloaded on the Federal Reserve Economic Data (FRED) website of the Federal Reserve Bank of St. Louis.[5] We compute the growth rate series as $y_t = (ipi_t - ipi_{t-1})/ipi_{t-1}$, where ipi_t is the IPI index at time t. The graphical representation of IPI growth rate appears in Figure 1, and reveals the presence of outliers.

In Panel A of Table 1, all detected jumps are given with their timing and t-statistics for both the GARCH and GJR-GARCH models. In addition, we also associate the date of each shocks to a specific event that occurred near that date.[6] Note that we find the same number and timing of shocks for the GARCH and GJR-GARCH models, except for three observations. Figure 2 displays the original and adjusted growth rates.

[4]Note that Inclán and Tiao (1994) advised that "*it is advisable to complement the search for variance changes with a procedure for outlier detection*".

[5]The IPI data had been seasonally adjusted by FRED prior to the analysis and are available at: https://fred.stlouisfed.org/series/INDPRO.

[6]Most of explanations associated to the detected shocks are found in the Federal Reserve Bulletin of the Board of Governors of the Federal Reserve System for the 1920-1997 period (https://fraser.stlouisfed.org) and in the Industrial Production and Capacity Utilization - G.17 of the Board of Governors of the Federal Reserve System for the 1997-2017 period (https://www.federalreserve.gov/releases/g17/).

Overall, most of (positive and negative) shocks can be associated with strikes in some industries, recessions, World War II and natural disasters, giving strong proof of infrequent large shocks.

Strikes. Industrial production declines in April 1927 (-2.4%) reflecting reduced activity both in mines and in factories due to the coal miners' strike. The large increase in March 1946 (+10.5%) is due to a sharp recovery in steel ingot production following settlement of the labor dispute. The decline in industrial production in October 1949 (-3.7%) is due to the steel strike. The increase in industrial production in August 1952 (+6.4%) can be explained by the increase in steel and iron ore production following settlement of the dispute (work stoppages) at steel works and iron ore mines in July. Further, the industrial production recovers after strikes in steel industry in August 1956 (+4.1%) and in December 1959 (+6.2%), and auto industry November 1964 (+3.1%).

Recessions. The fall in industrial production in April 1932 (-6.8%) can be explained by the Great Depression. Industrial production sharply declines in November and December 1937 (-9.8% and -8.9%, respectively) with a sharp reduction in the durable good industries, associated with the 1937-1938 recession. The decline in April 1980 (-2%) can be explained by the 1980 recession, implying reductions in the industrial production. The post WWI expansion (1919-1920) can explain the increase in industrial production in January 1920 (+9.4%) due to the strong rise in agricultural exports following the Victory loan boom.

World War II. Industrial activity increases in September 1939 (+6.1%) after the outbreak of war in Europe. The rise of industrial production in April 1942 (+2.8%) reflects continued advances in armament production, especially activity in the iron and steel, machinery, aviation, and shipbuilding industries, after the entering in war of the U.S. in December 1941. The strong decline in industrial activity in August and September 1945 (-10.4% and -8.9%, respectively) can be explained by the surrender of Japan, implying cancelation of military contracts, and thus decline in production in aircrafts, shipbuilding and ordnance plants. Industrial activity considerably increases in May and June 1933 (+16.6% and +15.3%, respectively) with the increased activity in the steel industry, reflecting the increased demand from automobile producers, associated with an advance in the general level of commodity prices.

Natural disasters. Industrial production falls after hurricanes Katrina and Rita in September 2005 (-1.8%) and hurricanes Gustav and Ike in September 2008 (-4.3%). These drops are also associated with strikes at a major aircraft producer. The reduction in motor vehicle output was responsible for a great part of the decline in industrial production (-1.4%) in January 1978 as well as severe storm activity over much of the US which caused widespread absenteeism, shorter workweeks, and some supply disruption.

Finally, the decline in industrial production in November 1974 (-3.3%) reflects a widening of cutbacks in production of consumer goods and materials, following the oil shock, and the effects of coal strike.

Table 2 presents summary statistics for the output growth variables, for both original and adjusted series (Panel A). As regards the original variables, empirical statistics indicate that the output growth rate is not Normally distributed (Jarque-Bera test). As regards higher moments of the distribution, output growth rate exhibits evidence of significant positive skewness and excess kurtosis. Blanchard and Simon (2001) note that the distribution of output growth exhibits excess kurtosis, if large and infrequent shocks occur. This suggests that the evidence of kurtosis may reflect extreme changes in growth rate. The Ljung-Box test leads to the presence of serial correlation in the series. The Lagrange Multiplier test for the presence of ARCH effects clearly indicates that output growth variable shows strong conditional heteroscedasticity.

Once outliers are accounted for, measures of non-Normality in adjusted series improve, sometimes quite dramatically, reducing excess skewness and excess kurtosis, but still without a Normal error structure because of excess kurtosis. This result confirms that of Fagiolo et al. (2008) who demonstrate that fat tails emerge in output growth rates independently of the length of time lags used to compute the growth rates and outliers. Finally, evidence of conditional heteroscedasticity is still found. From the comparison of basic statistics, it turns out that accounting for outliers diminishes deviation to Normality, which is an expected result. However, this does not prevent from evidence of ARCH effects at this stage.

The time periods of a shift in volatility as detected by the modified ICSS algorithm from the adjusted data are given in Panel B of Table 1. The results show the presence of three variance changes, with a first shift in October 1946 associated with the end of the WWII, a second in February 1962, and finally a variance change in January 1984.[7] This last break in volatility is associated with the well documented decline in output growth volatility in the eighties, characterized in the literature as the "Great Moderation" period. This break date estimated is in line with that found by McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), Stock and Watson (2003), Cecchetti et al. (2006), and Charles et al. (2018), among others.[8] Figure 3 displays the variance changes.

We also display summary statistics for the (outlier-adjusted) output growth rate in

[7]Note that the studies on the U.S. quarterly GNP/GNP growth exhibit only one structural break during the post-WWII period, around 1984Q2 (see, e.g., Stock and Watson, 2005; Summers, 2005; Cecchetti et al., 2006; Fang et al., 2014; Charles et al., 2018).

[8]Among the potential factors of this Great Moderation period, the literature put forward (i) 'good practices', i.e.: improved inventory management (e.g., McConnell and Perez-Quiros, 2000); (ii) 'good policies', i.e.: good monetary policy (e.g., Clarida et al., 2000; Bernanke, 2004; Boivin and Giannoni, 2006; Gali and Gambetti, 2009); and (iii) 'good luck', i.e.: a decline in the volatility of exogenous shocks (e.g., Stock and Watson, 2003, 2005; Ahmed et al., 2004).

four subperiods following the detected variance changes (Panel B of Table 2). The results show that the volatility of industrial production declines along the subperiods, where the pre-WWII period (1919-1946) is the highest volatile period, with a standard deviation five times that of the Great Moderation period (1984-2017), defined as the lowest volatile period, with 3.30 and 0.61, respectively. Due to the fact that the subperiod 1984-2017 includes the global financial crisis (GFC) we exclude this period by decomposing the subperiod 1984-2017 into two subperiods: 1984-2007 and 2010-2017. We find that the Great Moderation period (1984-2007) and the aftermath period of the GFC (2010-2017) are well the lowest volatile periods, with the latter displaying the smallest standard deviation (0.47%). All the sub-samples are characterized by non-Normality, especially the 1984-2017 subperiod. However, when excluding the GFC the non-Normality measures dramatically decrease, especially the skewness (1984-2007) or the kurtosis (2010-2017) which become significant. An interesting result is that the 1946-1962, 1962-1983, 1984-2007 and 2010-2017 subperiods do not exhibit evidence of ARCH effect, suggesting that the GARCH and GARCH-M modelling could not be appropriate for these subperiods.[9]

3 Output volatility modelling

In this section, we assess the impact on modeling of not taking outliers and variance changes into account. As argued by Fernández-Villaverde and Rubio-Ramírez (2010), modelling volatility is important to understand the source of aggregate fluctuations, the evolution of the economy, and for policy analysis. Further, it is necessary to have an accurate modeling of volatility to propose structural models with mechanisms

[9]We wish to thank the referee for pointing out that the GFC should not be considered as a part of the Great Moderation, and thus should be excluded from the last subperiod. The results show the impact of the GFC on the summary statistics.

that generate it (Fernández-Villaverde and Rubio-Ramírez, 2007, 2010; Justiniano and Primiceri, 2008). In this respect, we estimate AR(p)-GARCH(1,1) and AR(p)-GJR-GARCH(1,1) models for the growth rate series on two datasets: (1) raw data and (2) outlier-adjusted data. Indeed, GARCH-type models have proved useful in the measurement of output volatility in the empirical literature.

The conditional mean growth rate is supposed to follow an AR(p) process of the form:

$$x_t = \phi_0 + \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t, \tag{14}$$

where the lag order p is selected from the Schwarz Bayesian criterion (SBC) in order to capture growth dynamics and to produce uncorrelated residuals. For all t, $x_t = y_t$ for the original series or $x_t = \tilde{y}_t$ for the adjusted series, with

$$\begin{aligned} &\varepsilon_t = v_t \sqrt{\sigma_t^2}, \\ &\varepsilon_t \sim N(0, \sqrt{\sigma_t^2}), \qquad v_t \sim i.i.d.N(0,1), \\ &\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \beta\sigma_{t-1}^2 \end{aligned} \tag{15}$$

The parameters should satisfy $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ to guarantee the positivity of the conditional variance. The stationarity of the process (the second-order moment condition) is guaranteed by the restriction $\alpha + \beta < 1$. Ling and McAleer (2002a, 2002b) derived the regularity conditions of a GARCH(1,1) model as follows: $E[\varepsilon_t^2] = \frac{\omega}{1-\alpha-\beta} < \infty$ if $\alpha+\beta < 1$, and $E[\varepsilon_t^4] < \infty$ if $k\alpha^2 + 2\alpha\beta + \beta^2 < 1$, where k is the conditional fourth moment of z_t.[10] Ng and McAleer (2004) underline the importance of checking these conditions.

The sum of α and β quantifies the persistence of shocks to conditional variance, meaning that the effect of a volatility shock vanishes over time at an exponential rate. The GARCH models are short-term memory which define explicitly an intertemporal

[10]Under the assumption of a Normal distribution $k = 3$, so the condition becomes $3\alpha^2 + 2\alpha\beta + \beta^2 < 1$.

causal dependence based on a past time path. It is possible to shed light on the speed of the mean reversion process from GARCH parameters, based on the half-life concept. Half-life gives the point estimate of half-life (j) in quarters given as $(\alpha+\beta)^j = \frac{1}{2}$, so the half-life is given by $j = ln(0.5)/ln(\alpha+\beta)$, i.e. it takes for half of the expected reversion back towards $E(\sigma^2)$ to occur (Andersen and Bollerslev, 1997). When $\alpha+\beta = 1$ an Integrated GARCH (IGARCH) model is defined (Engle and Bollerslev, 1986), for which the unconditional variance is not finite, implying that the shocks to the conditional variance indefinitely persist.

In order to take into account the possible presence of asymmetry effect we also consider the GJR-GARCH model developed by Glosten, Jagannnathan et Runkle (1993). Specification for the conditional variance of GJR-GARCH(1,1) model is

$$\begin{aligned} \sigma_t^2 &= \omega + \alpha\, \varepsilon_{t-1}^2 + \gamma\, I(\varepsilon_{t-1})\, \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 \\ &= \omega + (\alpha + \gamma\, I(\varepsilon_{t-1}))\; \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 \end{aligned} \tag{16}$$

where $I(\varepsilon_{t-1}) = 1$ if $\varepsilon_{t-1} < 0$, and 0 otherwise. The volatility is positive if $\omega > 0$, $\alpha > 0$, $\gamma \geq 0$, $\alpha+\gamma \geq 0$ and $\beta \geq 0$. The process is defined as stationary if the constraint $\alpha+\beta+(\gamma/2) < 1$ is satisfied. Ling and McAleer (2002b) have derived the regularity conditions for a GJR-GARCH(1,1), defined as follows: $E[\varepsilon_t^2] < \infty$ if $\alpha+\beta+\delta\gamma < 1$, and $E[\varepsilon_t^4] < \infty$ if $k\alpha^2 + 2\alpha\beta + \beta^2 + \beta\gamma + k\alpha\gamma + k\delta\gamma^2 < 1$.[11]

Asymmetry exists if $\gamma > 0$, i.e. positive and negative shocks of equal magnitude have different effects on conditional volatility. The asymmetry is observed as the impulse $(\alpha+\gamma)$ of negative shocks, which is larger than the impulse (α) of positive shocks. In this model, good news and bad news have different effects on the conditional variance: good news has an impact of α while bad news has an impact of $(\alpha+\gamma)$. The GJR-GARCH model nets the GARCH model when $\gamma = 0$.[12]

[11] Under a Normal distribution $\delta = \frac{1}{2}$.

[12] Trypsteen (2017) also use a GJR-GARCH model to capture the asymmetric effect whereas Fang et

Tables 3 and 4 provide the estimation results for the AR(p)-GARCH(1,1) and AR(p)-GJR-GARCH(1,1) models on the full sample and the sub-samples, respectively. The parameters of the volatility models are estimated by maximizing the (quasi) log-likelihood function from the quasi-Newton method of Broyden, Fletcher, Goldfarb and Shanno (BFGS) and the standard-errors estimates are obtained from the outer product of the gradients estimator. To estimate the GARCH models, we use the package G@RCH 8.0 for Ox. We comment below the results for the full sample and the sub-samples.

Full sample. For the original data (y_t) the conditions of stationarity ($E[\varepsilon_t^2] < \infty$) and existence of the fourth moment ($E[\varepsilon_t^4] < \infty$) are not satisfied for the GARCH and GJR-GARCH models (Table 3). When shocks are taken into account ($\tilde{y}_t$) the condition of stationarity is satisfied for the GARCH model, implying that this condition can be biased by the presence of jumps, but the existence of the fourth moment is still not satisfied. This finding confirms that of Ng and McAleer (2004), showing that outliers can affect the moment conditions of GARCH models. The GJR-GARCH model does not satisfy the regulatory conditions. These results suggest that estimating the volatility models on a long period are not appropriate to capture the time-varying volatility of the output growth rate, even if the outliers are taken into account. Note that the residual tests show the presence of non-normality and autocorrelation but the ARCH effect is well taken into account.

Sub-samples ((outlier-adjusted data). For the pre-WWII period (1919-1946) the both volatility models do not satisfy the regularity condition, suggesting that the GARCH

al. (2014) apply an EGARCH model.

and GJR-GARCH models are not appropriate to model this high volatile period. The estimates of GARCH and GJR-GARCH parameters are non-significant for the subperiods 1946-1962 and 1962-1983, implying no (G)ARCH effect during these periods. This finding was expected for the subperiod 1946-1962 as the LM-ARCH test has rejected the presence of ARCH effects. Therefore, we cannot estimate GARCH-M models for both subperiods, suggesting there is no relationship between output volatility and its growth. Finally, the volatility of output growth for the subperiod 1984-2017 is appropriately modeled by an ARCH(1) model and exhibits a very low volatility persistence, with a persistence estimate of 0.256 and a half-life of shocks to volatility almost half a month. When excluding the GFC the ARCH(1) parameter stays significant for the two subperiods 1984-2007 and 2010-2017. Note that there is no asymmetric effect, whatever the subperiods, suggesting that negative shocks have not different effect on the variance than positive shocks in the IPI growth rate. The residual tests show that there is no ARCH effect for all the subperiods as well as no autocorrelation, except for the subperiods 1984-2017 and 2010-2017. These results suggest the importance of the ARCH modelling for the industrial production. However, the residual tests for the subperiods 1984-2017 and 2010-2017 display the presence of non-normality and autocorrelation, implying that the estimation of the ARCH(1) model should be taken with caution for these both subperiods.

4 The growth-volatility relationship

In this section, we analyze the growth-volatility relationship by applying a GARCH-in-Mean model (Engle et al., 1987). The mean growth rate is defined as:

$$x_t = \phi_0 + \sum_{i=1}^{p} \phi_i x_{t-i} + \lambda\sigma_t + \varepsilon_t \tag{17}$$

where for all t, $x_t = y_t$ for the original series or $x_t = \tilde{y}_t$ for the adjusted series, where

σ_t equals the standard deviation of the conditional variance (σ_t^2) defined in equations (15) or (16) for the GARCH and GJR-GARCH models, respectively, and λ measures the amplitude of the volatility effect.[13]

Full sample. Table 3 displays the GARCH-in-Mean estimation results for the original and adjusted data. For the original data, we find a significant relationship between output volatility and its growth from only the GARCH(1,1)-in-Mean model, with a positive relationship ($\lambda > 0$). However, the stationary condition is not satisfied for the GARCH-in-Mean and GJR-GARCH-in-Mean models. When outliers are taken into account, the growth-volatility relationship is again significant from the GARCH(1,1)-in-Mean model but the existence of the fourth moment is not satisfied, as for the GJR-GARCH-in-Mean model.

Sub-samples ((outlier-adjusted data). The GARCH-in-Mean estimations for the sub-periods 1919-1946, 1984-2017 and 1984-2007 are given in Table 5. The GARCH-in-Mean models are not estimated for the subperiods 1946-1962 and 1962-1983 since no ARCH effect has been detected, and for the subperiod 2010-2017 because the ARCH-M estimation fails to converge.[14] For the pre-WWII subperiod we find no relationship between output volatility and its growth, whatever the GARCH-in-Mean model. For the 1984-2017 subperiod we have evidence of a significant and positive relationship between output volatility and its growth from the ARCH(1)-in-Mean model ($\lambda > 0$) but exhibiting non-normality and autocorrelation in the residuals. However, when excluding the GFC the ARCH-M parameter becomes not significant, suggesting the effect

[13] As in Fang and Miller (2008) and Fang et al. (2008), we have also introduced lagged output growth into the conditional variance equation in the GARCH-in-Mean model to avoid potential endogeneity bias but the results are not significant.

[14] This problem can be explained by the very low number of observations to estimate an ARCH-M model (less than 100 observations).

of the crisis period on the ARCH-M modelling and that there is no growth-volatility relationship during the Great Moderation.

Overall, empirical results presented in this section underline that we find no relationship between output volatility and its growth during the full sample 1919-2017 and also for all the subperiods. From a macroeconomic point of view, this implies that economic performances, as measured by IPI growth, do not depend on the uncertainty as measured by IPI volatility.[15]

5 Robustness check

As a robustness check we repeat the estimations on one other proxy for output, by keeping a monthly frequency. For that we take the Chicago Fed National Activity Index (CFNAI) which is a monthly index designed to describe the economic activity.[16] The CFNAI is a weighted average of 85 monthly indicators of the U.S. economic activity, based on the work of Stock and Watson (1999).[17]

First, we also find the same outliers than those detected for industrial production in the pre-1967 period, namely in November 1974, January 1978 and April 1980

[15]Empirical evidence of the growth-volatility relationship is mixed: negative (Henry and Olekalns, 2002), positive (Fountas and Karanasos, 2006; Fang and Miller, 2014), or no statistically significant relationship (Grier and Perry, 2000; Fang et al., 2008). The disagreements can be explained by the differences in terms of the time period examined, the frequency of the data, and the methodology employed.

[16]To have a better fit for the GARCH models we focus on macroeconomic data with a high frequency. The correlation between industrial production and CFNAI is of 0.86 on the period 1967-2017.

[17]The economic indicators are drawn from four broad categories of data: (1) production and income; (2) employment, unemployment, and hours; (3) personal consumption and housing; and (4) sales, orders, and inventories. The derived index provides a single, summary measure of a factor common to these economic data.

(Figure 4), except the two outliers associated with hurricanes (September 2005 and 2008). This result suggests that the industrial production has been more affected by these natural disasters than the whole of the economic activity. The modified ICSS algorithm identifies only one shift in volatility in June 1984, associated with the Great Moderation. This break in CFNAI volatility is detected five months after the break found in industrial production.

From this result we define subperiods following the detected variance change, namely 1967-1984 and 1984-2017. For the last subperiod, we also split it into two subperiods by excluding the GFC, as for the industrial production: 1984-2007 and 2010-2017.

Table 6) displays the summary statistics for the full sample (raw and adjusted data, Panel A) and the subperiods (Panel B). The results show that the full period as well as all the subperiods exhibit non-Normality, autocorrelation and ARCH effect, except the 2010-2017 subperiod.

Table 7 provides the estimation results for the GARCH and GARCH-M models on the full sample for the raw and adjusted data. For the both data the volatility of CFNAI is better modelled by a symmetric GARCH model because either the condition of the fourth moment is not satisfied for the GJR model (raw data) or the asymmetric parameter is not significant (adjusted data). Further, when outliers are taken into account, the GARCH parameter on the lagged conditional variance (β) becomes not significant, showing again the effect of the outliers on the GARCH modelling. Finally, we find no evidence of a growth-volatility relationship in the full sample as for industrial production.

The results for GARCH models in the subperiods are given in Table 8 and are similar to those found for industrial production. The (G)ARCH model is not appropriate during the pre-1984 subperiod for CFNAI since all the parameters in the variance equation are not significant. Only the subperiod before the GFC (1984-2007) appears to be well modelled by an ARCH(1) model. Therefore, we cannot estimate

GARCH-M models for the subperiods 1967-1984 and 2010-2017.

The subperiod 1984-2017 seems to exhibit a significant and positive relationship between output volatility and its growth, as found for industrial production. However, when excluding the GFC we find again no evidence of output-growth relationship for CFNAI. This result confirms those found for industrial production and show the spurious effect of the GFC on the estimation of the ARCH-M models.

6 Conclusion

In this paper, we investigated the relationship between output volatility and growth using the standard GARCH-in-Mean framework and the U.S. monthly industrial production index (IPI) for the period January 1919 to December 2017. We analyzed the growth-volatility relationship in three ways. First, in addition to the GARCH-in-Mean model we also estimated a GJR-GARCH-in-Mean model where the variance equation includes a nonlinear term that allows negative shocks to have a different effect on the variance than positive shocks. Second, we employed the semi-parametric procedure proposed by Laurent, Lecourt and Palm (2016) to detect large shocks that affect the IPI growth, and try to associate these shocks to political, financial or economic events. Third, we identified shifts in volatility to define low volatility periods and high volatility periods. The detected shocks and variance changes are then taken into account for examining both volatility modelling and growth-volatility relationship.

There are three main conclusions that can be drawn from the analysis. First, IPI growth is strongly affected by (positive and negative) large shocks which are associated with strikes in some industries, recessions, World War II and natural disasters. We also identify several subperiods with different level of volatility where the volatility declines along the subperiods, with the pre-WWII period (1919-1946) the highest volatile period and the aftermath period of the GFC (2010-2017) the lowest volatile period. Second, accounting for jumps and variance changes are important to uncover the relationship between output volatility and growth, whereas the asymmetric GARCH effects is not significant. Third, the results show no evidence of relationship between output volatility and its growth during the full sample 1919-2017 and also for all the subperiods. From a macroeconomic point of view, this implies that economic performances, as measured by IPI growth, do not depend on the uncertainty as measured by IPI volatility.

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Figure 1: Growth rate of industrial production (1919-2017)

Figure 2: Raw and adjusted series of industrial production (1919-2017)

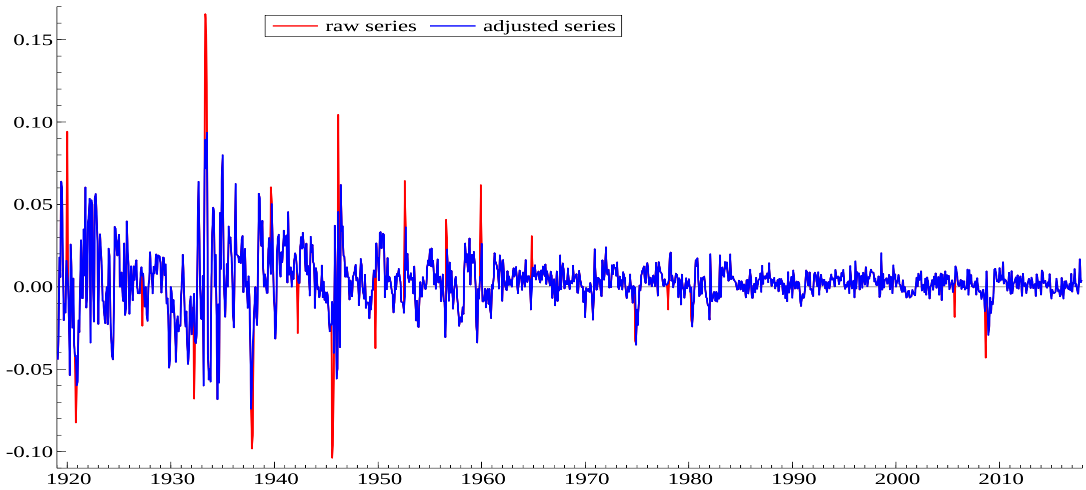

Figure 3: Variance changes of industrial production (1919-2017)

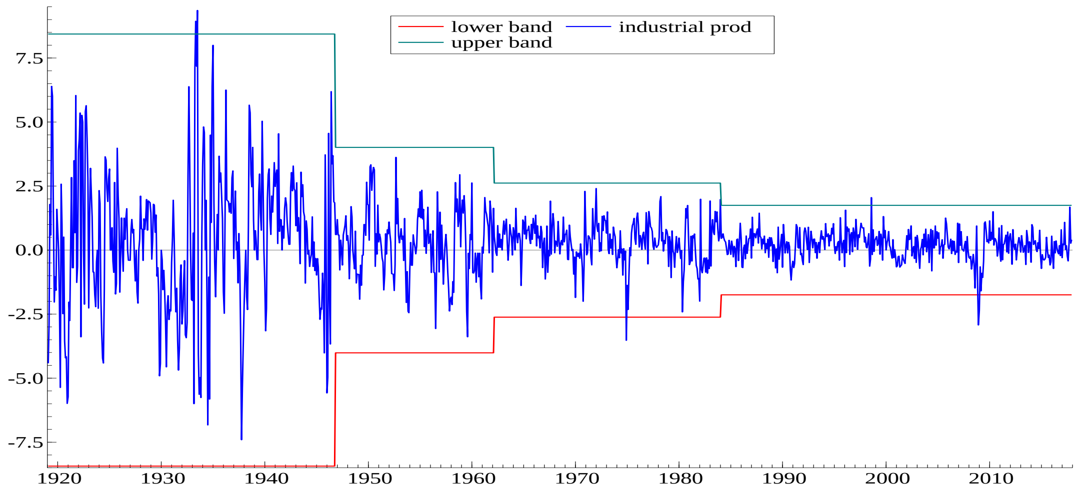

Figure 4: Raw and adjusted series of CFNAI (1919-2017)

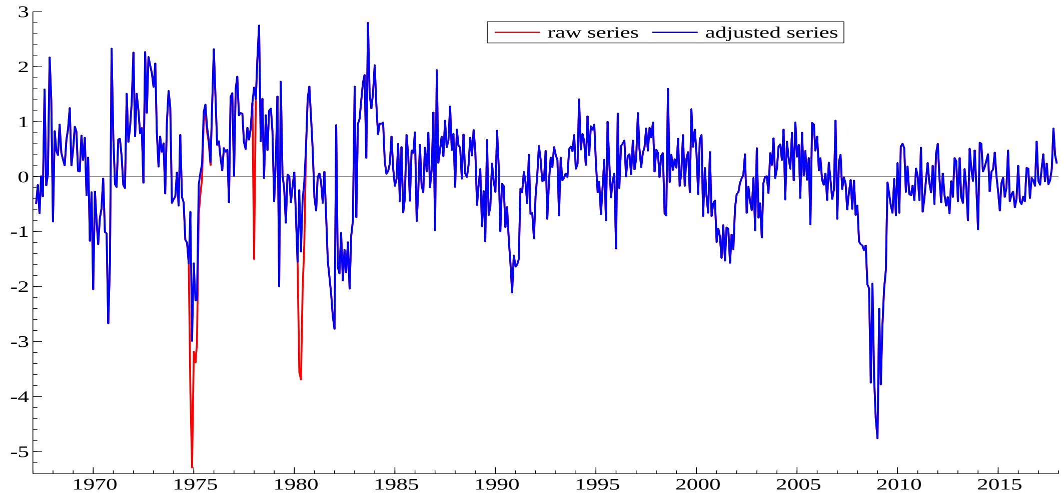

Table 1: Jumps and variance changes detected in the IPI growth.

Date	GARCH t-stat	GJR t-stat	Percent change	Events
Panel A: Jumps				
1920M1	4.098	4.440	9.42	Victory loan boom
1920M11	-4.442	-4.687	-8.24	
1927M4	-3.792	-4.092	-2.37	Coal miners' strike
1932M4	-4.018	–	-6.79	Great Depression
1933M5	6.214	6.521	16.56	strong steel production
1933M6	6.358	6.658	15.34	strong steel production
1937M11	-4.317	-4.571	-9.82	Reduction of durable good industries
1937M12	-4.261	-4.453	-8.87	Reduction of durable good industries
1939M9	3.778	4.712	6.05	Outbreak of war in Europe
1942M4	3.690	4.468	2.81	World War II
1945M8	-6.662	-5.355	-10.38	Surrender of Japan
1945M9	-3.445	–	-8.94	Surrender of Japan
1946M3	5.683	5.650	10.45	End of strikes
1949M10	-5.332	-5.048	-3.73	Steel strike
1952M8	7.146	5.959	6.43	Increase in steel and iron ore production
1956M8	3.621	–	4.08	End of steel strike
1959M12	4.182	3.744	6.18	End of steel strike
1964M11	4.223	4.044	3.09	End of auto industry strike
1974M11	-4.629	-4.075	-3.29	Oil shock
1978M1	-3.697	-3.807	-1.38	Reduction in auto production
1980M4	-3.786	-3.478	-2.04	Recession
2005M9	-4.030	-3.750	-1.84	Hurricanes Katrina & Rita
2008M9	-4.439	-3.465	-4.31	Hurricanes Gustav & Ike
Panel B: Variance changes				
1946M10				World War II
1962M2				
1984M1				Great Moderation

Notes: The Gumbel critical value with $T = 1187$ and $\alpha = 0.5$ is 3.44798.

Table 2: Descriptive Statistics for industrial production.

Series	Mean (%)	St. dev. (%)	Skewness	Excess Kurtosis	Jarque-Bera	$Q(12)$	$LM(12)$
Panel A: Full sample							
raw data	0.276	1.922	0.672*	12.5*	7847.2*	469.2*	61.8*
adjusted data	0.279	1.644	0.211*	5.21*	1349.4*	491.8*	63.8*
Panel B: Sub-samples							
Adjusted data							
1919M2-1946M9	0.352	3.300	0.396*	3.22*	151.8*	149.7*	10.2*
1946M10-1962M1	0.356	1.506	0.552*	3.93*	38.0*	131.6*	2.62
1962M2-1983M12	0.288	0.881	-0.741*	2.39*	86.4*	85.7*	1.42
1984M1-2017M12	0.171	0.610	-1.440*	8.75*	1441.7*	138.6*	4.37*
1984M1-2007M12	0.235	0.517	-0.038	3.95*	11.3*	41.2*	10.8
2010M1-2017M12	0.178	0.466	0.546*	3.46	5.62	13.9	2.08

Notes: * and ** mean significant at 5% and 10% level, respectively. The Jarque-Bera test for normality is distributed as a $\chi^2(2)$. $Q(12)$ is the Ljung-Box statistic at lag 12 of the standardized residuals. It is asymptotically distributed as $\chi^2(k)$, where k is the lag length. $LM(12)$ is the ARCH LM test at lag 12. It is distributed as $\chi^2(q)$, where q is the lag length. For each test the lag k is the total length lag.

Table 3: Estimation results for GARCH and GARCH-M models on full sample for industrial production (1919-2017).

Series	raw data		adjusted data		raw data		adjusted data	
Models	GARCH[b]	GJR[b]	GARCH[a]	GJR[b]	GARCH-M[a]	GJR-M[a]	GARCH-M[a]	GJR-M[a]
ϕ_0	0.003 (8.36)	0.002 (3.78)	0.003 (6.13)	0.002 (4.45)	0.001* (1.57)	0.001* (0.88)	0.001* (1.19)	0.001* (1.25)
ϕ_1	0.171 (7.59)	0.280 (9.83)	0.304 (10.7)	0.310 (10.9)	0.174 (6.76)	0.287 (9.37)	0.300 (10.2)	0.313 (10.7)
ϕ_2	0.142 (6.09)	0.107 (3.74)	0.119 (3.94)	0.130 (4.31)	0.150 (6.18)	0.110 (3.81)	0.120 (3.92)	0.130 (4.32)
ϕ_3	0.156 (6.56)	0.114 (4.34)	0.103 (3.58)	0.105 (3.94)	0.154 (6.46)	0.113 (4.20)	0.103 (3.56)	0.104 (3.89)
λ					0.279 (3.37)	0.158* (1.54)	0.253 (1.97)	0.138* (1.08)
ω^c	0.063 (6.51)	0.018 (4.42)	0.015 (4.20)	0.012 (3.50)	0.063 (6.80)	0.021 (4.56)	0.015 (4.25)	0.014 (3.51)
α	0.455 (11.8)	0.051 (3.70)	0.127 (8.99)	0.053 (4.10)	0.451 (12.3)	0.051 (3.61)	0.126 (9.07)	0.053 (4.05)
β	0.610 (24.2)	0.819 (75.0)	0.864 (67.9)	0.866 (72.1)	0.609 (25.8)	0.815 (75.8)	0.866 (69.3)	0.866 (70.8)
γ	–	0.330 (10.9)	–	0.168 (7.11)	–	0.318 (10.2)	–	0.161 (6.92)
persist.	1.065	1.035	0.992	1.003	1.060	1.025	0.991	0.999
half-life	–	–	86.3	–	–	–	79.3	518.1
LL	3676.7	3710.6	3795.7	3816.0	3682.5	3711.8	3797.7	3816.6
Residual tests								
Jarque-Bera	585.3*	496.8*	320.8*	201.0*	479.6*	487.0*	314.4*	204.6*
$Q(12)$	31.6*	15.2*	17.7*	15.0*	31.6*	16.0*	18.3*	15.3*
$LM(12)$	1.04	1.19	0.57	0.48	0.89	1.12	0.66	0.52

Notes: $(\alpha+\beta)$ and $(\alpha+\beta+\gamma/2)$ measure the volatility persistence for the GARCH and GJR-GARCH models, respectively. Half-life gives the point estimate of half-life (j) in days given as $persist.^j = \frac{1}{2}$. The ARCH-M models are defined as: $y_t = \phi_0 + \sum_{i=1}^{4} \phi_i y_{t-i} + \lambda\sigma_t + \varepsilon_t$; and $\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2 + \theta y_{t-1}$. [a] denotes that the condition for existence of the fourth moment of the model is not observed. [b] denotes that the stationarity condition of the model is not satisfied. [c] the constant in variance ω is multiplied by 10^4. The t-statistics are given in parentheses. * means significant at 5% level.

Table 4: Estimation results for GARCH models in sub-samples for industrial production (adjusted data).

Series	1919-1946		1946-1962		1962-1983		1984-2017		1984-2007		2010-2017	
Models	GARCH[a]	GJR[a]	GARCH	GJR	GARCH	GJR	GARCH	GJR	GARCH	GJR	GARCH	GJR
ϕ_0	0.006 (2.76)	0.005 (2.00)	0.003* (1.42)	0.003* (1.36)	0.003 (3.83)	0.003 (3.86)	0.002 (4.77)	0.002 (4.34)	0.002 (5.32)	0.002 (4.80)	0.002 (5.43)	–
ϕ_1	0.558 (11.8)	0.529 (16.4)	0.559 (8.96)	0.557 (8.84)	0.361 (5.38)	0.349 (5.18)	0.110 (2.30)	0.081 (1.94)	−0.035* (−0.55)	−0.015* (−0.23)	0.033* (0.59)	–
ϕ_2	–	–	–	–	–	–	0.180 (3.81)	0.169 (3.69)	0.253 (4.39)	0.228 (3.93)	0.188 (3.98)	–
ϕ_3	–	–	–	–	–	–	0.174 (3.75)	0.175 (4.02)	0.210 (3.86)	0.200 (3.64)	0.185 (3.80)	–
ω^c	0.374 (2.15)	2.842 (9.11)	0.389 (0.52)	0.389 (0.57)	0.556 (15.9)	0.548 (15.9)	0.194 (11.8)	0.193 (11.8)	16.5 (8.49)	17.2 (8.48)	0.002 (1.92)	–
α	0.323 (4.27)	0.360 (3.13)	0.060* (0.60)	0.041* (0.41)	0.030* (0.68)	0.019* (0.32)	0.256 (4.55)	0.040* (0.81)	0.251 (2.67)	0.062* (0.61)	0.122 (1.78)	–
β	0.645 (8.80)	–	0.612* (0.88)	0.603* (0.94)	–	–	–	–	–	–	–	–
γ	–	0.529 (1.99)	–	0.060* (0.45)	–	0.057* (0.66)	–	0.446 (4.02)	–	0.29* (1.86)	–	–
persist.	0.967	0.624	–	–	–	–	0.256	–	0.251	–	0.122	–
half-life	20.8	1.5	–	–	–	–	0.51	–	0.50	–	0.33	–
LL	810.8	795.2	572.5	572.7	911.3	911.4	1586.9	–	1142.7	–	1599.3	–
Jarque-Bera	6.08*	7.63*	5.17	4.76	99.0*	101.0*	28.0*	11.9*	1.49	0.88	16.6*	–
$Q(12)$	9.62	10.6	5.34	5.50	14.4	15.2	23.5*	21.8*	12.6	12.5	18.5*	–
$LM(12)$	0.63	0.48	0.30	0.27	1.11	1.15	1.11	0.97	0.75	0.68	1.47	–

Notes: $(\alpha+\beta)$ and $(\alpha+\beta+\gamma/2)$ measure the volatility persistence for the GARCH and GJR-GARCH models, respectively. Half-life gives the point estimate of half-life (j) in days given as $persist.^j = \frac{1}{2}$. [a] denotes that the condition for existence of the fourth moment of the GARCH is not observed. [b] denotes that an IGARCH model has been estimated because the stationarity condition of the GARCH model were not satisfied. [c] the constant in variance ω is multiplied by 10^4. The t-statistics are given in parentheses. * means significant at 5% level.

Table 5: Estimation results for GARCH-M models in sub-samples for industrial production (adjusted data).

Series	1919-1946		1984-2017		1984-2007	
Models	GARCH-M[a]	GJR-M[a]	GARCH-M	GJR-M	GARCH-M	GJR-M
ϕ_0	0.005 (0.98)	−0.001 (−0.14)	−0.001 (−0.45)	−0.001 (−0.67)	0.001* (0.17)	−0.001* (−0.22)
ϕ_1	0.561 (11.8)	0.550 (17.3)	0.098 (2.12)	0.142 (2.17)	−0.025* (−0.40)	0.025* (0.31)
ϕ_2	–	–	0.208 (4.29)	0.171 (3.48)	0.261 (4.71)	0.238 (4.11)
ϕ_3	–	–	0.183 (3.65)	0.166 (3.20)	0.209 (3.86)	0.186 (3.22)
λ	0.074* (0.28)	0.271* (1.44)	0.653 (1.83)	0.621 (1.89)	0.478* (1.03)	0.611* (1.25)
ω^c	0.377 (2.16)	2.747 (8.51)	0.187 (12.4)	0.188 (12.4)	16.5 (8.69)	17.1 (9.05)
α	0.322 (4.18)	0.412 (3.46)	0.286 (4.82)	0.047* (0.89)	0.248 (2.78)	0.069* (0.81)
β	0.644 (8.71)	–	–	–	–	–
γ	–	0.464* (1.62)	–	0.458 (3.84)	–	0.288 (1.94)
persist.	0.966	0.644	0.286	–	0.248	–
half-life	19.9	1.6	0.55	–	0.50	–
LL	810.8	795.3	1588.9	1595.2	11.43.1	–
Jarque-Bera	6.00*	7.58*	25.4*	12.5*	1.03	0.76
$Q(12)$	9.62	10.5	24.7*	21.1*	12.5	11.7
$LM(12)$	0.63	0.41	1.29	1.15	0.78	0.84

Notes: The ARCH-M models are defined as: $y_t = \phi_0 + \sum_{i=1}^{4} \phi_i y_{t-i} + \lambda\sigma_t + \varepsilon_t$; and $\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2 + \theta y_{t-1}$. [a] denotes that the condition for existence of the fourth moment of the GARCH is not observed. [b] denotes that the ARCH-M estimation fails to converge. [c] the constant in variance ω is multiplied by 10^4. The t-statistics are given in parentheses. * means significant at 5% level

Table 6: Descriptive Statistics for CFNAI (1967-2017).

Series	Mean	St. dev.	Skewness	Excess Kurtosis	Jarque-Bera	$Q(12)$	$LM(12)$
Panel A: Full sample							
raw data	0.002	1.016	-1.140*	6.75*	488.6*	1096.0*	43.2*
adjusted data	0.011	0.946	-1.876*	10.2*	1103.0*	1445.5*	50.3*
Panel B: Sub-samples							
Adjusted data							
1967M3-1984M6	0.195	1.159	-0.349*	-3.28	4.87	454.1*	3.66*
1984M7-2017M12	0.023	0.911	-1.440*	8.75*	1441.7*	976.0*	60.1*
1984M7-2007M12	0.052	0.649	-0.463*	3.23	10.7*	344.1*	6.59*
2010M1-2017M12	-0.045	0.383	0.111	2.19	2.19	12.9	0.52

Notes: * and ** mean significant at 5% and 10% level, respectively. The Jarque-Bera test for normality is distributed as a $\chi^2(2)$. $Q(12)$ is the Ljung-Box statistic at lag 12 of the standardized residuals. It is asymptotically distributed as $\chi^2(k)$, where k is the lag length. $LM(12)$ is the ARCH LM test at lag 12. It is distributed as $\chi^2(q)$, where q is the lag length. For each test the lag k is the total length lag.

Table 7: Estimation results for GARCH and GARCH-M models on full sample for FCNAI (1967-2017).

Series	raw data		adjusted data		raw data		adjusted data	
Models	GARCH	GJR[a]	GARCH	GJR	GARCH-M	GJR-M[a]	GARCH-M	GJR-M
ϕ_0	0.131* (1.91)	0.019* (0.24)	0.001* (0.01)	−0.018* (−0.14)	0.066* (0.51)	0.018* (0.13)	−0.171* (−1.05)	−0.282* (−1.63)
ϕ_1	0.201 (4.30)	0.226 (4.85)	0.209 (4.33)	0.212 (4.35)	0.204 (4.39)	0.226 (4.67)	0.206 (4.22)	0.240 (4.08)
ϕ_2	0.279 (6.11)	0.269 (6.27)	0.385 (10.3)	0.380 (9.97)	0.286 (6.00)	0.269 (6.20)	0.378 (10.4)	0.350 (7.76)
ϕ_3	0.186 (4.63)	0.188 (4.66)	0.218 (5.75)	0.214 (5.58)	0.184 (4.52)	0.188 (4.44)	0.214 (5.68)	0.187 (4.44)
λ	–	–	–	–	0.125* (0.61)	0.003* (0.012)	0.400* (1.66)	0.561 (2.52)
ω	0.124 (4.44)	0.112 (4.20)	0.295 (11.1)	0.297 (11.0)	0.132 (4.11)	0.113 (4.18)	0.292 (10.8)	0.296 (10.9)
α	0.414 (5.69)	0.161 (2.52)	0.329 (3.79)	0.266 (2.47)	0.428 (5.42)	0.161 (2.49)	0.340 (3.81)	0.204 (2.02)
β	0.388 (4.82)	0.452 (5.26)	–	–	0.363 (3.82)	0.452 (4.97)	–	–
γ	–	0.409 (3.58)	–	0.112 (0.81)	–	0.112* (0.81)	–	0.234* (1.63)
persist.	0.803	0.818	0.329	0.322	0.790	0.818	0.340	0.321
half-life	3.16	3.44	0.62	0.61	2.94	3.44	0.64	0.61
LL	-615.8	-607.8	-582.5	-582.2	-615.6	-607.8	-581.4	-580.1
Jarque-Bera	33.4*	23.8*	29.3*	31.0*	35.3*	29.3*	31.7*	32.0*
$Q(12)$	24.6*	18.5*	17.2*	17.2*	23.7*	18.1*	17.1*	17.7*
$LM(12)$	0.40	0.27	1.17	1.15	0.33	0.27	1.08	0.92

Notes: $(\alpha+\beta)$ and $(\alpha+\beta+\gamma/2)$ measure the volatility persistence for the GARCH and GJR-GARCH models, respectively. Half-life gives the point estimate of half-life (j) in days given as $persist.^j = \frac{1}{2}$. ^{a} denotes that the condition for existence of the fourth moment of the model is not observed. The t-statistics are given in parentheses. * means significant at 5% level. The best LL in-sample criteria is given in bold.

Table 8: Estimation results for GARCH models in sub-samples for CFNAI (adjusted data).

Series	1967-1984		1984-2017		1984-2007		2010-2017	
Models	GARCH[a]	GJR[a]	GARCH	GJR	GARCH	GJR[b]	GARCH	GJR
ϕ_0	0.226* (0.71)	0.227* (0.71)	−0.045* (−0.32)	−0.091* (−0.70)	0.065* (0.55)	–	−0.048* (−0.93)	−0.050* (−0.96)
ϕ_1	0.354 (4.74)	0.352 (4.61)	0.110 (1.99)	0.121 (2.13)	0.050* (0.83)	–	0.129* (1.25)	0.139* (1.30)
ϕ_2	0.332 (4.75)	0.337 (4.47)	0.404 (9.42)	0.388 (9.01)	0.371 (7.09)	–	−0.066* (−0.69)	−0.066* (−0.70)
ϕ_3	0.137 (1.98)	0.136 (1.97)	0.309 (6.42)	0.288 (5.87)	0.324 (5.93)	–	0.213 (2.12)	0.206 (2.02)
ω	0.589 (7.84)	0.585 (7.54)	0.217 (10.3)	0.222 (10.3)	0.219 (8.59)	–	0.121 (5.28)	0.120 (5.19)
α	0.072* (0.80)	0.093* (0.58)	0.226 (2.92)	0.039* (0.51)	0.202 (2.30)	–	0.063* (0.50)	0.035* (0.24)
β	–	–	–	–	–	–	–	–
γ	–	−0.029* (−0.17)	–	0.426 (2.93)	–	–	–	0.071* (0.32)
persist.	–	–	0.226	–	0.201	–	–	–
half-life	–	–	0.47	–	0.43	–	–	–
LL	-247.2	-247.1	-306.0	–	-213.2	–	–	–
Jarque-Bera		3.90	1.39	–	1.10	–	–	–
$Q(12)$		8.68	9.68	–	9.71	–	–	–
$LM(12)$		0.86	2.02*	–	1.05	–	–	–

Notes: $(\alpha+\beta)$ and $(\alpha+\beta+\gamma/2)$ measure the volatility persistence for the GARCH and GJR-GARCH models, respectively. Half-life gives the point estimate of half-life (j) in days given as $persist.^j = \frac{1}{2}$. The ARCH-M models are defined as: $y_t = \phi_0 + \sum_{i=1}^{4} \phi_i y_{t-i} + \lambda\sigma_t + \varepsilon_t$; and $\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2 + \theta y_{t-1}$. [a] denotes that the condition for existence of the fourth moment of the GARCH is not observed. [b] denotes that the ARCH-M estimation fails to converge. The t-statistics are given in parentheses. * means significant at 5% level.

Table 9: Estimation results for GARCH-M models in sub-samples for CFNAI (adjusted data).

Series	1984-2017		1984-2007	
Models	GARCH-M	GJR-M	GARCH-M	GJR-M
ϕ_0	-0.308^* (-1.70)	-0.392 (-2.24)	-0.221^* (-0.85)	-0.219^* (-0.79)
ϕ_1	0.107 (2.08)	0.169 (2.66)	0.052^* (0.86)	0.103^* (1.35)
ϕ_2	0.401 (9.40)	0.359 (6.98)	0.391 (7.45)	0.346 (5.57)
ϕ_3	0.309 (6.42)	0.288 (5.87)	0.299 (6.39)	0.246 (4.46)
λ	0.649 (2.35)	0.682 (2.38)	0.611^* (1.30)	0.514^* (0.99)
ω	0.209 (10.3)	0.217 (10.6)	0.218 (8.70)	0.227 (8.46)
α	0.249 (3.28)	0.051^* (0.69)	0.200 (2.36)	0.013^* (0.14)
β	–	–	–	–
γ	–	0.321 (2.45)	–	0.294^* (1.90)
persist.	0.249	0.212	0.200	–
half-life	0.50	0.45	0.43	–
LL	-302.8	-299.9	-212.3	–
Jarque-Bera	0.45	0.17	1.51	–
$Q(12)$	8.18	9.01	9.24	–
$LM(12)$	1.91^*	1.30	1.14	–

Notes: $(\alpha+\beta)$ and $(\alpha+\beta+\gamma/2)$ measure the volatility persistence for the GARCH and GJR-GARCH models, respectively. Half-life gives the point estimate of half-life (j) in days given as $persist.^j = \frac{1}{2}$. The ARCH-M models are defined as: $y_t = \phi_0 + \sum_{i=1}^{4} \phi_i y_{t-i} + \lambda\sigma_t + \varepsilon_t$; and $\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2 + \theta y_{t-1}$. ^{a} denotes that the condition for existence of the fourth moment of the GARCH is not observed. The t-statistics are given in parentheses. * means significant at 5% level.