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The Strategic Allocation to Style-Integrated Portfolios of Commodity Futures

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Abstract

Our study lies at the intersection of the literature on the diversification benefits of commodity futures and the literature on style integration. It augments the traditional asset mix of investors with a long-short portfolio that integrates the styles that matter to the pricing of commodity futures. Treating the style-integrated portfolio of commodities as part of the strategic mix of investors is found to enhance out-of-sample performance and reduce crash risk compared to the alternatives considered thus far. The conclusion holds across traditional asset mix, portfolio allocation methods, integration strategies, and sub-periods. The diversification benefits of style integration also persist, albeit lower, in a long-only setting.

Keywords: Commodity futures, Style integration, Strategic asset allocation, Diversification

JEL classifications: G11, G12, G14

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1. Introduction

Are investors better off adding commodities to their traditional portfolio of equities and bonds? Which commodity portfolio is optimal as part of the strategic mix? Which asset allocation technique designs a diversified portfolio with the best risk-adjusted performance and lowest crash risk? The literature has investigated these issues for decades and drawn various conclusions on i) the commodity portfolio that shall be considered as a building block of the strategic mix, and ii) the methodology employed to allocate wealth to the equity, bond, and commodity building blocks.

In-sample studies have concluded that commodities should be part of an investor's asset mix.[1] The conclusion holds irrespective of the choice of the commodity class (e.g., individual commodity futures, long-only and long-short commodity portfolios) and is unchallenged by the weighting scheme adopted to allocate wealth to the various asset classes (fixed weights, optimized weights, stochastic dominance weights, volatility timing weights). In-sample studies, however, suffer from the major drawback of assuming perfect knowledge of the assets' returns, risks and correlations through the entirety of the sample period. Clearly, such perfect foresight is unreasonable, and these studies can thus only give us an idea of the maximum diversification benefits that could have been obtained in the past. It follows that the conclusions drawn are of limited value to asset managers interested in designing practical investment solutions. Skiadopoulos (2012) concurred by stating that "there is a need to explore commodity investment vehicles that would make the investor better off in a real-time, out-of-sample setting."

Bearing this in mind, many studies have analyzed the benefits of diversification on an out-of-sample basis, this time assuming that the strategic mix defined at the end of a given

[1] Bodie and Rosansky (1980); Fortenbery and Hauser (1990); Ankrim and Hensel (1993); Anson (1999); Jensen et al. (2000, 2002); Daskalaki and Skiadopoulos (2011); Bessler and Wolff (2015); Daskalaki et al. (2017); Yan and Garcia (2017); Fethke and Prokopczuk (2018).

month is used to structure a diversified portfolio over the following month.[2] The literature is much more divided on the above questions. Benefits tend to be stronger when considering more sophisticated long-only and long-short commodity portfolios and tend to disappear when investors incorporate naive long-only commodity portfolios (such as the S&P-GSCI) into their strategic mix (Daskalaki and Skiadopoulos, 2011; Blitz and de Groot, 2014; Kremer, 2015; Lombardi and Ravazzolo, 2016; Yan and Garcia, 2017; Fethke and Prokopczuk, 2018). The method used to design the strategic mix also seems to matter. For example, i) asset allocations that maximize expected utility are often found to be sub-optimal relative to schemes based on fixed weights, volatility timing or stochastic dominance (Daskalaki and Skiadopoulos, 2011; Bessler and Wolff, 2015; Daskalaki et al., 2017) and ii) forward looking asset allocations that account for predictability beat backward-looking asset allocations that ignore it (Gao and Nardari, 2018).

We contribute to the above literature on commodity allocation by intersecting it with the literature on style integration. Style integration refers to forming a portfolio that buys commodities with strong buy recommendations across styles, sells commodities with strong sell recommendations across styles, and assigns near-zero weights to commodities with weak or conflicting signals.[3] This literature has established that such style-integrated portfolios offer better risk-adjusted performance and lower crash risk than long-only or single-style portfolios. These benefits accrue across asset classes such as equities (Brandt et al., 2009; Fischer and Gallmeyer, 2016), currencies (Kroencke et al., 2014; Barroso and Santa-Clara,

[2]Daskalaki and Skiadopoulos (2011); You and Daigler (2013); Blitz and de Groot (2014); Bessler and Wolff (2015); Kremer (2015); Lombardi and Ravazzolo (2016); Daskalaki et al. (2017); Yan and Garcia (2017); Fethke and Prokopczuk (2018); Gao and Nardari (2018).

[3]Let us assume that the style-integrated portfolio combines K signals. Let us assume further that half of the signals recommed buying commodity i, while the other half recommend selling it, and that all K signals recommend buying commodity j. Reflecting upon this, the integrated portfolio is likely to ignore commodity i (for which we have no strong buy or sell recommendation) and will be very tilted towards commodity j (for which we have K buy recommendations). The actual allocations assigned to commodities i and j reflect these simple principles and are detailed in Section 2.1.1.

2015) and on a cross-market analysis (Fernandez-Perez et al., 2019).

Since equally weighting the signals is the best style integration method (Fernandez-Perez et al., 2019), it is appropriate to revisit the conclusions on the diversification benefits of commodities using the integrated portfolio as a commodity building block in place of the commodity portfolios used thus far. By doing so, we consider whether the characteristics that govern the pricing of commodity futures such as carry, hedging pressure, momentum, value, or skewness can be used to design portfolios diversified across asset classes with enhanced out-of-sample performance.

There are three reasons why the commodity long-short style-integrated portfolio that we assess could be considered as a worthy candidate for inclusion into traditional portfolios of equities and fixed income securities. First, over the sample analysed, the portfolio presents a higher Sharpe ratio (1.02) than that of any of the building blocks considered. Second, it often has lower return correlations with equity and fixed income portfolios than any of the other commodity portfolios that we study. Third, its conditional correlations with equities are particularly low in periods of heightened volatility in equity markets. This finding is welcome: The risk diversification emanating from low conditional correlations seems to occur when most needed.

The diversification benefits of the style-integrated commodity portfolio are measured by comparing the out-of-sample risk-adjusted performance and crash risk profile of two portfolios: one that solely invests into stocks and bonds and another that adds the style-integrated commodity portfolio to the strategic mix. The analysis is conducted considering various versions of the integrated portfolio (long-only and long-short) as well as various fixed income

and equity building blocks.[4] Finally, in an effort to draw robust conclusions and following the literature on diversification, we consider various approaches to define the allocation to the equity, fixed income and commodity building blocks ranging from a naive equally-weighted scheme to more sophisticated optimization-based and volatility-timed approaches. Throughout the paper, the analysis is conducted on an out-of-sample basis; as such, it has practical implications for asset managers interested in style integration and diversification across asset classes.

We find that treating the long-short style-integrated commodity portfolio as a building block consistently provides diversification benefits to portfolios of stocks and bonds. On average, adding the long-short commodity integrated portfolio to the traditional asset mix of investors enhances the Sharpe ratio of the stock and bond portfolios by 61.9%. Regression analysis demonstrates that, by including the long-short integrated commodity portfolio in their traditional portfolios, investors earn positive and statistically significant alphas that average 2.42% a year across portfolios with a range from 0.84% (t-statistic of 4.46) to 3.36% (t-statistic of 4.92).

Albeit to a lower extent than with a long-short allocation, style integration adds value also in a long-only setting. This finding has significant implications for the field of portfolio management: It is reassuring to see that the benefits of diversification are not just open to managers with long-short mandates; those benefits are also available to managers who are restricted from taking short positions. Confirming previous findings (Daskalaki and Skiadopoulos, 2011; Fethke and Prokopczuk, 2018), we note a failure to enhance risk-adjusted performance when using naive long-only commodity portfolios (S&P-GSCI or a long-only

[4]The traditional building blocks of stocks and bonds include i) long positions in traditional indices (Barclays US Aggregate Bond Index and the S&P500 Index), ii) the five factors of Fama and French (2015) alongside the momentum factor of Carhart (1997), and iii) integrated portfolios of stock index futures and fixed income futures, whereby the latter portfolios combine many signals in a manner similar to the integration strategy used for commodity futures.

equally-weighted and monthly-rebalanced portfolio of commodity futures) as building blocks. The superiority of the strategic mix that allows for style integration holds irrespective of the stock and bond building blocks considered and across portfolio allocation methods. This conclusion is also robust to the consideration of a two-step approach to style integration and to the analysis of various sub-periods.

Kremer (2015) and Daskalaki et al. (2017) found that, unlike naive first-generation commodity indices, second- and third-generation commodity indices provide diversification benefits.[5] Fethke and Prokopczuk (2018) nuanced these results by noting that some third-generation commodity indices weaken the out-of-sample performance of the diversified portfolio, suggesting that they should be excluded from the strategic mix of investors. Our conclusions extend this literature by showing that asset managers should, first and foremost, construct well-designed long-only or long-short integrated portfolios and, then and only then, allocate wealth to these efficient building blocks.

The remainder of the article is structured as follows. Section 2 describes the methodology used, focusing in particular on the integration approach and on the techniques used to allocate wealth to the building blocks. Section 3 details the data and sampling. Section 4 presents the results and highlights the diversification benefits of style integration. Finally, Section 5 concludes.

[5]Miffre (2014) defined i) first generation commodity indices as naive, broad-based indices of long-only commodity futures (e.g., S&P-GSCI), ii) second generation indices as long-only indices that capitalize on backwardation and/or mitigate the harmful impact of contango on performance, and iii) third generation indices as structured products that buy commodity futures deemed to appreciate in value (e.g., backwardated assets) and short commodity futures deemed to depreciate in value (e.g., contangoed assets). Miffre (2014) argued that third-generation indices outperform their first and second-generation counterparts, as they recognize the fundamentals of commodity futures pricing (see, e.g., Miffre (2016), for a review).

2. Methodology

This section details how the integrated portfolios are formed (Section 2.1), defines the portfolios of stocks, bonds, and commodities used as building blocks (Section 2.2), presents the techniques used to allocate wealth to these building blocks (Section 2.3), and describes the methodologies employed to measure the benefits of diversification (Section 2.4).

2.1. Forming the integrated portfolios

2.1.1 Integration approach

Following Fernandez-Perez et al. (2019), the allocation of the integrated portfolio to the N commodity futures present at the time of portfolio formation, Φ_t, is given by the following equation

$$\Phi_t = \Theta_t \times \Omega_t = \begin{pmatrix} \theta_{1,1,t} & \dots & \theta_{1,K,t} \\ \vdots & \ddots & \vdots \\ \theta_{N,1,t} & \dots & \theta_{N,K,t} \end{pmatrix} \times \begin{pmatrix} \omega_{1,t} \\ \vdots \\ \omega_{K,t} \end{pmatrix} \tag{1}$$

$i = \{1, \dots, N\}$ denotes the cross section of available commodity futures, $t = \{1, \dots, T\}$ denotes the portfolio formation time, and $k = \{1, \dots, K\}$ denotes a signal. Ω_t is a $K \times 1$ vector of weights that defines the allocation of the integrated portfolio to each of the K single-style portfolios. Following Fernandez-Perez et al. (2019), we assign equal weights to each of the K styles and thus, $\omega_{k,t} = 1/K$. Θ_t is a $N \times K$ matrix with elements $\theta_{i,k,t}$ that determines the weight of the ith futures contract in the kth single-style portfolio at time t. Following Brandt et al. (2009) or Barroso and Santa-Clara (2015), $\theta_{i,k,t}$ equals the characteristic or signal cross-sectionally standardized, namely

$$\theta_{i,k,t} = (y_{i,k,t} - \bar{y}_{k,t})/\sigma_{k,t} \tag{2}$$

where $y_{i,k,t}$ is the value of signal k for futures contract i at time t with cross-sectional mean and standard deviation denoted as $\bar{y}_{k,t}$ and $\sigma_{k,t}$, respectively. A positive $\theta_{i,k,t}$ indicates a long (L) position and a negative $\theta_{i,k,t}$ indicates a short (S) position. The integrated portfolio is fully collateralized and thus, the allocation to the N assets defined in Equation (1) is normalized to ensure full investment. Mathematically,

$$\tilde{\phi}_{i,t} = \frac{\phi_{i,t}}{\sum_{i=1}^{N} |\phi_{i,t}|} \tag{3}$$

to ensure $\sum_{i=1}^{N} \tilde{\phi}_{i,t} = 1$. Given the cross-sectional standardization of the individual signals and the full collateralization of the integrated portfolio, $\sum_{i=1}^{N_L} \tilde{\phi}_{i,t} = \sum_{i=1}^{N_S} \tilde{\phi}_{i,t} = 0.5$ with $N_L + N_S = N$. The resulting portfolio is long-short in nature and is thereafter labelled as the long-short integrated risk premium (LS-IRP) portfolio.

Aside from the LS-IRP portfolio, we also consider a long-only integrated risk premium (LO-IRP) portfolio that solely buys commodity futures that are deemed to appreciate according to the integrated strategy. The allocation to the N_L futures contracts is then defined as $\tilde{\phi}^L_{i,t}$, where $\tilde{\phi}^L_{i,t} = \frac{\phi^L_{i,t}}{\sum_{i=1}^{N_L} |\phi^L_{i,t}|}$, to ensure full collateralization. We consider this long-only portfolio to reflect upon i) the preference of investors for long-only portfolios, ii) the long-only restrictions that are often imposed on the mandates of institutional asset managers, and iii) a recent literature that studies the diversification benefits of well-designed long-only commodity portfolios based on e.g. momentum (Blitz and de Groot, 2014; Daskalaki et al., 2017; Yan and Garcia, 2017).

To avoid look-ahead bias, the construction of the long-short integrated risk premium (LS-IRP) portfolio and of the long-only integrated risk premium (LO-IRP) portfolio is conducted on an out-of-sample basis: Each style-integrated portfolio formed at the end of month t is held for a month, at which time another style-integrated portfolio is formed, and so forth, until the sample ends. We address concerns pertaining to transaction costs by systematically considering as part of our strategic asset allocation the excess returns of the IRP portfolios

net of transaction costs, $r_{IRP,t+1}$, which we measure as follows:

$$r_{IRP,t+1} = \sum_{i=1}^{N} \tilde{\phi}_{i,t} r_{i,t+1} - \sum_{i=1}^{N} TC|\tilde{\phi}_{i,t+1} - \tilde{\phi}_{i,t}(1 + r_{i,t+1})| \tag{4}$$

where $r_{i,t+1}$ is the excess return of the i^{th} commodity between t and $t+1$, TC is a conservative level of transaction costs equal to 8.6 b.p. (Marshall et al., 2012), and $\tilde{\phi}_{i,t}(1 + r_{i,t+1})$ is the actual portfolio weight right before the next rebalancing time $t + 1$.

2.1.2 Integrated signals

The literature on the pricing of commodity futures contracts has identified a large set of characteristics or styles that explain the time series and the cross section of commodity futures prices. It is important to note that our objective here is not to define the integrated portfolio with the best out-of-sample performance but rather to demonstrate the benefits of diversification given a chosen integrated portfolio. Thus, we do not claim to be exhaustive in our choice of signals and have simply selected the five signals that Fernandez-Perez et al. (2019) used in their study of cross-market integration (carry, momentum, value, skewness, liquidity) alongside three extra signals that are commodity-specific (hedging pressure, basis-momentum, relative basis).

The signals, $y_{i,k,t}$ for $k = \{1, \ldots, 8\}$ in Equation (2), are such that higher values indicate longer positions. They are constructed as follows:

- **Carry.** Carry is defined as the slope of the term structure of commodity futures prices. Mathematically, $y_{i,1,t} = ln(f^1_{i,t}) - ln(f^2_{i,t})$, where $f^k_{i,t}$ denotes the futures price with location k on the term structure of commodity i at time t. The use of carry as a signal for asset allocation emanates from the theory of storage of Kaldor (1939), Working (1949), and Brennan (1958). The theory of storage asserts that a positive carry indicates backwardation due to scarcity, while a negative carry indicates contango due to abundance. In turn, backwardation (contango) predicts an appreciation (depreciation)

in the futures price in the short run (Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006; Szymanowska et al., 2014). Koijen et al. (2018) extended the carry signal to markets other than commodities.

- **Momentum.** Momentum is defined as the mean of the daily returns of front-end futures contracts, $r_{i,t}$, over the prior year. Mathematically, $y_{i,2,t} = \frac{1}{252}\sum_{j=0}^{251} r_{i,t-j}$. Asness et al. (2013) showed that recent winners outperform recent losers across markets.

- **Value.** The value signal is measured as the ratio of the front-end futures price averaged over a period spanning 4.5 to 5.5 years before portfolio formation to the front-end futures price at the time of portfolio formation. Namely, we have $y_{i,3,t} = \frac{\frac{1}{D}\sum_{d=0}^{D-1} f^1_{i,t-d}}{f^1_{i,t}}$ where D is the number of trading days in the 4.5 to 5.5 years before t (Asness et al., 2013). Assets with high value signals are currently underpriced and thus expected to outperform. Again, this signal is priced across markets.

- **Skewness.** The skewness signal is defined as the negative of the third central moment of the return distribution over the year prior to the formation of the portfolio. Mathematically, $y_{i,4,t} = -[\frac{1}{252}\sum_{d=0}^{251}(r_{i,t-d} - \mu_{i,t})^3]/\sigma^3_{i,t}$, where $\mu_{i,t}$ and $\sigma_{i,t}$ denote the mean and standard deviation, respectively, of daily front-end returns over the previous year. The minus sign is here to denote that we buy contracts with higher $y_{i,4,t}$ signals and presumably negative skewness and short contracts with lower $y_{i,4,t}$ signals and presumably positive skewness. Evidence in favor of the pricing of skewness can be found across markets; see, for example, Amaya et al. (2015) for equities, Chiang (2016) for fixed income securities, and Fernandez-Perez et al. (2018) for commodity futures.

- **Liquidity.** The liquidity signal, based on Amihud (2002) illiquidity measure, is defined as $y_{i,5,t} = \frac{1}{D}\sum_{d=0}^{D-1} \frac{|r_{i,t-d}|}{\$Volume_{i,t-d}}$, where D is the number of days over the previous two months. This signal also applies across markets; see, for example, Amihud et al. (2015) for equities and fixed income securities and Szymanowska et al. (2014) for commodities.

- **Hedging pressure.** The hedging pressure signal is defined as the net short positions of large hedgers relative to their total position as averaged over the weeks of the prior year. Formally, $y_{i,6,t} = \frac{1}{52}\sum_{w=0}^{51} \frac{Short_{i,t-w} - Long_{i,t-w}}{Short_{i,t-w} + Long_{i,t-w}}$, where $Short_{i,w}$ and $Long_{i,w}$ are the week w short and long open interests of large hedgers (i.e., commercial traders) on commodity futures i. The use of hedging pressure as a signal for asset allocation emanates from the hedging pressure hypothesis of Cootner (1960) and Hirshleifer (1989). The theory asserts that net short hedging indicates backwardation, while net long hedging indicates contango. Evidence in support of the theory can be found in, for example, Bessembinder (1992) and Basu and Miffre (2013).

- **Basis momentum.** The basis momentum signal is defined as the difference between the 252-day averaged returns of the front- and second-nearest contracts. Mathematically, $y_{i,7,t} = Mom^1_{i,t} - Mom^2_{i,t}$, where $Mom^j_{i,t}$ denotes the momentum signal based on the j^{th} futures returns. Higher values of the basis momentum signal predict higher excess returns in the near future. The signal is deemed to capture imbalances in the supply and demand of futures contracts during episodes of high volatility and illiquidity (Boons and Prado, 2019).

- **Relative basis.** This relative basis signal, defined as $y_{i,8,t} = ln(f^1_t) - 2ln(f^2_t) + ln(f^3_t)$, measures the curvature of the futures price curve. Positive values of the relative value signal indicate backwardation (due to high convenience yield and scarcity) and predict higher excess returns in the near future (Gu et al., 2021).

2.2. Building blocks

Asset allocation is based on various building blocks of equities, fixed income securities, and commodities. Details on each building block follow.

We consider three equity building blocks. The first building block uses the widely used and readily accessible S&P500 total return index. The second is a fully collateralized portfolio

of equity index futures that integrates the five signals that are pervasive across futures classes, namely carry, momentum, value, skewness, and liquidity. The method employed to form (integrate) these signals is the same as the one presented in Section 2.1.2 (Section 2.1.1). The total returns of the equity-based integrated portfolio are measured net of transaction costs using Equation (4). The third equity building block equally weights and rebalances on a monthly basis equity risk premia based on the market, size, book-to-market value, past performance, operating profitability, and investment (Fama and French, 1993; Carhart, 1997; Fama and French, 2015). The second and third equity building blocks are considered either in a long-only or a long-short setting.

We consider two fixed income building blocks: the widely used and readily accessible Barclays US Aggregate Bond Index and a style-integrated and fully collateralized portfolio of fixed income and interest rate futures. The latter portfolio, formed using the method of Section 2.1.1, combines the carry, momentum, value, skewness, and liquidity signals in a long-only or a long-short setting (Fernandez-Perez et al., 2019) and assumes round-trip transaction costs of 8.6 b.p., as in Equation (4).

Finally, we consider two types of commodity building blocks. The first set naively assumes long-only positions as proxied by the S&P-GSCI or a long-only, equally-weighted and monthly-rebalanced portfolio of commodity futures, hereafter referred to as the average portfolio (AVG). The second set uses the integrated approach of Section 2.1.1 in a long-only or a long-short setting and takes into account the eight signals detailed in Section 2.1.2. The excess returns of the AVG, LO-IRP commodity portfolio and LS-IRP commodity portfolio are measured net of transaction costs.

The use of futures contracts for the style-integrated building blocks naturally follows from considerations such as their high liquidity, low transaction cost, and absence of short-selling restrictions. Moreover, given the cost of carry model, futures positions can be considered as good substitutes to spot positions in the absence of arbitrage.

2.3. Portfolio construction

How much wealth should we allocate to these J building blocks, where $J = 2$ for the traditional portfolio made of stocks and bonds and $J = 3$ for the diversified portfolio that also treats commodities as part of the strategic mix? We use three methods of portfolio construction: equally-weighted (EW), optimization-based (OB), and volatility timing (VT).[6]

2.3.1 Equally-weighted portfolio

At the beginning of each month t, portfolio weights are naively assumed to be the same across assets; i.e., $X_t = (\frac{1}{J}, \ldots, \frac{1}{J})'$. The weights do not depend on any parameter aside from the number of asset classes. The approach is appealing for many reasons: It is very simple to implement, is free from estimation errors (since there is no parameter to estimate), and does not suffer from perfect foresight bias (since we do not assume that the past performance or past volatility of a given building block at time t will repeat itself at time $t + 1$). Equal-weighting schemes have been shown to improve the robustness of out-of-sample results in various contexts, such as asset allocation (DeMiguel et al., 2009), forecasts combination (Rapach et al., 2010), or style integration (Fernandez-Perez et al., 2019).

2.3.2 Optimization-based portfolio

The weights assigned to the J building blocks at time t, $x_{j,t}$ for $j = 1, \ldots, J$, are estimated by maximizing the conditional expected power utility of the portfolio's total returns at time $t + 1$. Mathematically, we have

$$\max_{x_{j,t}} E_t \left[U(R_{P,t+1})\right] = \max_{x_{j,t}} E_t \left[U\left(\sum_{j=1}^{J} x_{j,t} R_{j,t+1}\right)\right] = \max_{x_{j,t}} E_t \left[\frac{\left(1 + \sum_{j=1}^{J} x_{j,t} R_{j,t+1}\right)^{1-\gamma} - 1}{1-\gamma}\right] \tag{5}$$

[6]The main conclusion from the paper is that the average increases in Sharpe ratios obtained when adding the commodity LO or LS-IRP portfolio to the traditional strategic mix are substantial (at 27.6% and 61.9%, respectively). It is unlikely that these sizeable rises in performance will be wiped off by the costs of rebalancing the building blocks, and thus we assume that these transaction costs can be ignored.

where $R_{P,t+1}$ is the total return of the traditional (or diversified) portfolio at time $t+1$, $R_{j,t}$ is the total return of building block j at time t, and γ is the coefficient of relative risk aversion that we set equal to $\gamma = 3$.[7] We impose the following constraints: $\sum_{j=1}^{J} x_{j,t} = 1$, $0 \leq x_{E,t} \leq 0.6$, $0 \leq x_{FI,t} \leq 0.4$, and $0 \leq x_{C,t} \leq 0.3$, where E, FI, and C stand for the equity, fixed income, and commodity building blocks, respectively. The first constraint ensures full collateralization; the others are there to avoid corner solutions defined as close-to-boundary weights (Black and Litterman, 1992). These constraints enhance diversification (Grauer and Shen, 2000), reduce estimation errors (Frost and Savarino, 1988), and reflect the operational boundaries under which asset managers operate in practice.

The strategic mix of the traditional and diversified portfolios are obtained on an out-of-sample basis. We use a moving window of 60 months to estimate the optimal allocation to the building blocks and capture the performance of the traditional and diversified portfolios over the following month. The estimation window is then moved forward by one month and the portfolio formation process is then repeated.

2.3.3 Volatility timing portfolio

Portfolio allocation methods based on utility optimization suffer from a range of issues including extreme weights, high turnover, and poor out-of-sample performance (Jagannathan and Ma, 2003; Michaud, 2008; Kirby and Ostdiek, 2012; Low et al., 2016). Following, for example, Kirby and Ostdiek (2012) or Bessler and Wolff (2015), we weight each building block in an inverse relationship to the one-year variance of its daily returns. The volatility

[7] We use a power utility function because it satisfies standard investors' preferences (monotonicity and risk-aversion) and allows for the distribution of building block returns to depart from normality. We obtained similar conclusions on the benefits of diversification from the use of other utility functions, including negative exponential and power utility with disappointment aversion, or from assuming higher levels of risk aversion $\gamma = \{5, 10\}$.

timing (VT) weight for building block j at time t, $x_{j,t}$, is given by

$$x_{j,t} = \frac{1/\sigma_{j,t}^2}{\sum_{j=1}^{J} 1/\sigma_{j,t}^2} \tag{6}$$

where $\sigma_{j,t}$ is the volatility of the daily total return of building block j over the previous year. The allocations to the building blocks measured at t are used to form the VT portfolio for a month. The process is reiterated using rolling windows until the sample ends.

2.4. Measuring the diversification benefits of commodity exposure

2.4.1 Comparison of performance and risk profiles

A direct and simple way to assess the value of commodities in multi-asset portfolios is by comparing the performance and risk profile of traditional portfolios (i.e., constructed from equity and fixed income) with those of diversified portfolios (i.e., constructed by augmenting traditional portfolios with commodity exposures). Performance and risk profiles are measured and compared using various statistical metrics such as annualized mean, annualized volatility, skewness, and excess kurtosis of excess returns as calculated from standard mathematical formulas. Portfolio metrics include downside volatility (measured as annualized volatility of negative excess returns), 99% Cornish-Fisher Value-at-Risk (defined as maximum expected loss of the portfolio with 99% confidence after accounting for the third and fourth moments of the distribution of its returns), the percentage of months with positive excess returns, the portfolio's maximum drawdown, Sharpe ratio (defined as annualized mean excess return to annualized total volatility), Sortino ratio (defined as annualized mean excess return to annualized downside volatility), and Omega ratio (defined as probability of gains to probability of losses).

In line with DeMiguel et al. (2009), we test whether the Sharpe ratios of the diversified portfolios that include commodities are statistically different from their no-commodity counterparts using the approach of Jobson and Korkie (1981) after applying the correction

of Memmel (2003). Specifically, given portfolios a and b, with their estimated means, variances, and covariances over a sample size T represented by μ_a, μ_b, σ_a, σ_b, $\sigma_{a,b}$, the hypothesis $H_0 : \mu_a/\sigma_a - \mu_b/\sigma_b = 0$ is tested via the asymptotically normally distributed test statistic z:

$$\begin{aligned} z &= \frac{\sigma_b \mu_a - \sigma_a \mu_b}{\sqrt{\nu}}, \\ \nu &= \frac{1}{T}(2\sigma_a^2\sigma_b^2 - 2\sigma_a\sigma_b\sigma_{a,b} + \frac{1}{2}\mu_a^2\sigma_b^2 + \frac{1}{2}\mu_b^2\sigma_a^2 - \frac{\mu_a\mu_b}{\sigma_a\sigma_b}\sigma_{a,b}) \end{aligned} \tag{7}$$

Following Gao and Nardari (2018), we utilize the CRRA utility-based certainty equivalent excess return (CEQ) to further evaluate the performance of the diversified portfolios. CEQ is defined as the riskless return an investor with a CRRA utility preference is willing to accept to forgo the uncertain returns of a given strategy.[8] Specifically, let $\bar{U}_{1:T}(W_t)$ be the average realized CRRA utility for period $t = 1 \cdots T$. Certainty equivalent excess return is given by:

$$CEQ_{1:T} = [(1-\gamma)\bar{U}_{1:T}(W_t)]^{\frac{1}{1-\gamma}} - 1, \tag{8}$$

where W_t is investor's wealth at time t. Without loss of generality, we normalize initial wealth to 1, so we have $W_t = 1 + R_t$, where R_t is the strategy's excess return between $t-1$ and t.

To further evaluate if commodities add value to a traditional portfolio, we regress the excess returns of the diversified portfolio that includes commodities $r_{D,t}$ onto the excess returns of the corresponding traditional portfolio that excludes commodities $r_{T,t}$. Formally

$$r_{D,t} = \alpha + \beta r_{T,t} + \epsilon_{1,t} \tag{9}$$

where $\epsilon_{1,t}$ is a time t error term. A failure to reject the null hypothesis $H_0 : \alpha = 0$ will lead to the conclusion that the addition of commodities does not provide any incremental benefit

[8] CEQ is a useful performance measure, since it does not suffer from the shortcomings of other measures, such as Sharpe ratio, which can lead to misleading conclusions once the underlying assumptions of identically and normally distributed returns are violated.

to the traditional portfolio. Conversely, a positive and significant α will be interpreted as an excess return that is linked to the introduction of commodities and earned over and above that of the traditional asset mix.

2.4.2 Conditional correlation

There has been ample evidence of heightened correlation among securities and asset classes during times of increased volatility and market downturns.[9] This feature is concerning, as it is precisely during periods of increased equity volatility that investors need the benefits of diversification the most. We study whether the benefits of diversification, in terms of higher time-varying correlations between the traditional and commodity building blocks, are jeopardized when the volatility of traditional assets rises. Appendix A details the methods employed to measure conditional volatilities and conditional correlations as estimated via the generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1986) and the dynamic conditional correlation (DCC) model of Engle (2002), respectively.

We analyze the relationship between the conditional correlations between the traditional and commodity building blocks (i and j) and the conditional volatilities of the traditional building block i by estimating the following regression via OLS:

$$\rho_{ij,t} = \beta_0 + \beta_1 \sqrt{h_{i,t}} + \epsilon_{2,t} \tag{10}$$

where $\rho_{ij,t}$ is the time t conditional return correlation between asset classes i and j, $\sqrt{h_{i,t}}$ is the time t conditional volatility of asset class i, and $\epsilon_{2,t}$ is a time t error term. A positive and statistically significant β_1 coefficient signifies rising correlations in periods of increased volatility in traditional asset markets, and thus potentially lower diversification benefits. Conversely, a negative and statistically significant β_1 coefficient indicates lower correlations during market downturns. This latter case would be welcome news because the benefits of

[9] See Longin and Solnik (1995), Ang and Chen (2002), or Low et al. (2013).

diversification are most needed at such times.

3. Data and sampling

3.1. Commodity futures contracts

The commodities dataset, sourced from Refinitiv Datastream, covers 28 commodity futures. These comprise five metal futures (copper, gold, palladium, platinum, silver), four livestock futures (feeder cattle, frozen pork bellies, lean hogs, live cattle), six energy futures (electricity, gasoline, heating oil, light sweet crude oil, natural gas, unleaded gasoline), and 12 agricultural futures (cocoa, coffee, corn, cotton, frozen concentrated orange juice, oats, rough rice, soybeans, soybean meal, soybean oil, sugar number 11, wheat), alongside futures on lumber.

To alleviate liquidity issues, all price-based signals and simple excess returns[10] are based on front-end settlement prices except in maturity months, when next-nearest prices are used instead. When second-end futures prices are needed, the rolling to third contracts is timed so that it matches the rolling of front contracts. The hedging pressure signal is calculated using the open interest of large commercial traders (hedgers), as published in the Commitment of Traders' (COT) report by the Commodity Futures Trading Commission. All series are sampled at a daily frequency except for the open interests of hedgers, which are weekly.

Given the availability of COT data, the portfolio construction, and the signals definition mentioned in Section 2, the out-of-sample sample that is common to all portfolios spans the period from October 1998 to December 2018. The full sample period starts 10 years, as we need an initial five year period of data to form the value signal and a subsequent five year period of data to obtain the first out-of-sample optimized weights (see Equation (5)).

[10]Excess of the risk-free rate on fully collateralized positions.

3.2. Traditional assets

All returns for traditional assets are calculated as simple returns. The total returns of the S&P500 index and Barclays US Aggregate Bond index are obtained from Refinitiv Datastream and Bloomberg, respectively. We download from Refinitiv Datastream the data required to form integrated portfolios of stock index futures and fixed income and interest rate futures. Appendix B provides details on the underlying assets. The rolling from one contract to the next follows the same approach as that employed for commodities.

The total returns of long-only size, value, winner, operating profitability and conservative investment portfolios and the total returns of long-short size (SMB, Small-Minus-Big), value (HML, High-Minus-Low), momentum (WML, Winner-Minus-Loser), profitability (RMW, Robust-Minus-Weak) and investment (CMA, Conservative-Minus-Aggressive) portfolios are downloaded from Kenneth French's website. To form the corresponding long-only (long-short) equity building block, we equally weight and monthly rebalance the long-only (long-short) size, value, momentum, profitability and investment portfolios. This approach has been adopted to ensure that we consistently have one equity building block in the various traditional and diversified portfolios that we consider. The resulting long-only and long-short characteristic-sorted portfolios are denoted hereafter as LO-FF and LS-FF, respectively, in reference to the seminal contributions of Fama and French.

4. Empirical results

4.1. Performance and risk of the building blocks

Table 1 reports summary statistics of the excess returns of the equity, fixed income, and commodity building blocks over a period spanning from October 1998 to December 2018. The table highlights the good performance of long-only and long-short equity and fixed income portfolios. The noteworthy portfolios are the long-only FF portfolio and Barclays bond index, whose Sharpe ratios are at 0.60 and 0.75, respectively. The other traditional portfolios

present weaker performance, but we consider them nonetheless, as our objective here is not to implement profitable investment strategies but rather to study the diversification benefits obtained when adding style-integrated commodity portfolios to a given traditional asset mix.

The right-hand side of Table 1 confirms the conclusion of Fernandez-Perez et al. (2019) regarding the performance of the long-short integrated risk premia portfolio (LS-IRP) commodity portfolio: It presents a positive and statistically significant mean excess return (9.05% a year with a t-statistic of 3.66), low volatility compared to equity and other commodity portfolios (at 9% a year) and the highest Sharpe (1.02), Sortino (2.25), and Omega (2.22) ratios. These properties make the LS-IRP portfolio a worthy candidate for addition to the traditional portfolios. The long-only integrated risk premia portfolio (LO-IRP) commodity portfolio also performs well with an annualized mean excess return of 11.15% (t-statistic of 2.44), but its volatility is twice that of the LS-IRP commodity portfolio and, as a result, its Sharpe ratio is noticeably lower (0.63).

Table 1: Summary statistics of the building block portfolios
The table reports summary statistics for LO (long-only) and LS (long-short) portfolios. IRP corresponds to the integrated risk premium portfolio. FF refers to the characteristic-sorted portfolios of Carhart (1997) and Fama and French (2015). AVG is a long-only equally-weighted and monthly-rebalanced portfolio of commodity futures. Mean is the annualized average return in excess of the risk-free rate. Volatility and downside volatility are annualized. JB p-value refers to the p-value of the Jarque-Bera normality test with the null hypothesis that the sample comes from a normal distribution. VaR is the monthly Cornish-Fisher Value-at-Risk. Sharpe (Sortino) ratios are annualized mean excess return over annualized volatility (annualized downside volatility using 0% as the threshold). Omega ratios are the probability of gains divided by the probability of losses using 0% as the threshold. Bold fonts denote significance at the 5% level or better. t-statistics are in parentheses. The sample spans the period from October 1993 to December 2018.

	Equity					*Fixed income*			*Commodities*			
	S&P 500	LO IRP	LS IRP	LO FF	LS FF	Barclays	LO IRP	LS IRP	S&P GSCI	AVG	LO IRP	LS IRP
Mean	**0.0740**	**0.0765**	0.0632	**0.0934**	0.0068	**0.0262**	**0.0292**	0.0093	-0.0029	0.0215	**0.1115**	**0.0905**
	(2.01)	(2.21)	(1.85)	(2.46)	(0.50)	(2.80)	(2.10)	(1.56)	(-0.06)	(0.79)	(2.44)	(3.66)
Volatility	0.14	0.15	0.14	0.15	0.05	0.04	0.06	0.03	0.21	0.12	0.18	0.09
Downside volatility	0.11	0.10	0.11	0.12	0.04	0.02	0.05	0.02	0.15	0.09	0.10	0.04
Skewness	**-0.70**	0.07	**-0.37**	**-0.75**	**0.53**	-0.19	-0.23	**-0.42**	**-0.33**	**-0.44**	**0.65**	**0.90**
	(-4.94)	(0.50)	(-2.60)	(-5.33)	(3.77)	(-1.33)	(-1.64)	(-2.98)	(-2.35)	(-3.12)	(4.64)	(6.37)
Ex. kurtosis	**1.28**	**1.91**	**6.69**	**1.35**	**6.49**	**0.93**	**2.79**	**2.53**	**1.28**	**2.83**	**3.80**	**2.92**
	(4.55)	(6.77)	(23.77)	(4.79)	(23.04)	(3.30)	(9.93)	(8.98)	(4.56)	(10.04)	(13.52)	(10.38)
JB p.value	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
99%VaR (Cornish-Fisher)	-0.09	-0.13	-0.15	-0.09	-0.06	-0.03	-0.05	-0.02	-0.14	-0.09	-0.19	-0.09
% of positive months	0.64	0.58	0.59	0.62	0.47	0.61	0.56	0.52	0.53	0.53	0.58	0.60
Max drawdown	0.26	0.32	0.39	0.27	0.13	0.07	0.13	0.06	0.45	0.29	0.38	0.20
Sharpe ratio	0.51	0.51	0.46	0.60	0.13	0.75	0.47	0.32	-0.01	0.18	0.63	1.02
Sortino ratio (0%)	0.68	0.80	0.56	0.80	0.19	1.15	0.63	0.43	-0.02	0.25	1.09	2.25
Omega ratio (0%)	1.47	1.47	1.52	1.55	1.12	1.74	1.49	1.30	0.99	1.15	1.64	2.22

To get a sense of the diversification benefits of adding commodities to a traditional asset

mix, Panel A of Table 2 presents unconditional return correlations between i) the equity and fixed income building blocks and ii) the commodity building blocks. The table shows that the unconditional correlations range from -0.11 to 0.34 over the sample analysed, with an average at 0.08. This result confirms the wisdom that commodities serve as a diversification tool to traditional assets (Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006). The evidence is stronger for the LS-IRP commodity portfolio, whose correlations with equity portfolios are statistically indistinguishable from zero. Other things being equal, the LS-IRP commodity portfolio is therefore potentially better suited than the long-only commodity portfolios at diversifying equity risk. All the returns correlations with the Barclays bond index are statistically insignificant; thus, none of the commodity portfolios stands out as a better diversifier of the risks present in the bond market.

Panel B of Table 2 analyses the relationship between conditional correlations and the conditional volatilities of traditional assets and reports estimates of the slope coefficient, β_1, of Equation (10). Studying the conditional correlations with long-only equity portfolios (the S&P-500 index, equity-based LO-IRP portfolio, long-only FF portfolio), we note that the S&P-GSCI, AVG portfolio and LO-IRP commodity portfolio exhibit large β_1 estimates that are often positive at the 5% level or better and that average 1.44 across regressions. In other words, increases in equity volatility go hand-in-hand with increases in correlation. In periods of heightened volatility in equity markets (known as "risk-off environments"), a flight to quality takes place, such that asset managers then shift their strategic allocation away from risky assets (equities, long-only commodity futures) and toward low-risk assets (fixed income securities). These selling pressures lead to drops in both equity and commodity prices and therefore to an increase in their conditional correlations.

In sharp contrast, the slope coefficient β_1 obtained for the LS-IRP commodity portfolio vis-à-vis the volatility of long-only equity indices is equal to zero in statistical terms. This suggests that the LS-IRP commodity portfolio exhibits the same level of diversification re-

Table 2: Correlation analysis

Panel A reports unconditional return correlations between traditional and commodity building blocks, where the traditional building blocks are made of either equities or fixed income securities. p-value is for the null that the correlation is zero. Panel B presents slope coefficients obtained from regressions of conditional correlations between traditional and commodity building blocks onto the conditional volatilities of traditional building blocks. t-statistics in parentheses are adjusted for heteroskedasticity and autocorrelation by the Newey and West (1987) method with 12 lags. IRP corresponds to the integrated risk premium portfolio. FF refers to the characteristic-sorted portfolios of Carhart (1997) and Fama and French (2015). LO and LS stand for long-only and long-short portfolios, respectively. AVG is a long-only equally-weighted and monthly-rebalanced portfolio of commodity futures. The sample spans the period from October 1993 to December 2018.

	Commodity building blocks							
	S&P GSCI		AVG		LO IRP		LS IRP	
	Correlation	p-value	Correlation	p-value	Correlation	p-value	Correlation	p-value
Panel A: Unconditional correlation with commodity building blocks								
Equity building blocks								
S&P P5500	**0.26**	0.00	**0.34**	0.00	**0.23**	0.00	-0.01	0.86
LO IRP	**0.16**	0.01	**0.22**	0.00	**0.17**	0.00	0.02	0.79
LS IRP	**0.12**	0.05	**0.16**	0.01	**0.12**	0.03	0.02	0.75
LO FF	**0.31**	0.00	**0.34**	0.00	**0.24**	0.00	0.00	0.96
LS FF	0.00	0.96	-0.11	0.07	-0.09	0.12	-0.03	0.64
Fixed income building blocks								
Barclays	-0.02	0.71	0.03	0.55	0.03	0.63	-0.03	0.58
LO IRP	-0.05	0.37	-0.04	0.52	-0.00	0.95	-0.00	0.96
LS IRP	0.01	0.87	0.01	0.84	0.04	0.51	0.01	0.84
	β	t-stat	β	t-stat	β	t-stat	β	t-stat
Panel B: Conditional correlation with commodity building blocks								
Equity building blocks								
S&P 500	3.78	(1.60)	**1.98**	(2.17)	**2.69**	(2.04)	0.01	(0.28)
LO IRP	-0.00	(-0.46)	-2.22	(-1.14)	0.00	(0.23)	0.00	(0.78)
LS IRP	**0.69**	(4.27)	**0.63**	(3.98)	**0.66**	(4.03)	**0.25**	(4.15)
LO FF	**2.08**	(2.78)	**1.86**	(2.27)	**2.75**	(2.79)	0.04	(1.06)
LS IRP	0.05	(0.17)	-0.00	(-0.49)	-0.71	(-0.50)	1.20	(0.80)
Fixed income building blocks								
Barclays	0.41	(1.68)	12.35	(1.57)	7.72	(0.81)	1.69	(0.27)
LO IRP	**7.98**	(2.07)	3.06	(1.61)	2.96	(0.83)	0.16	(0.21)
LS IRP	3.52	(1.19)	0.20	(0.05)	-2.44	(-0.32)	-1.34	(-0.37)

gardless of the state of the equity market. This attribute is desirable because it indicates that investors get diversification in the form of low conditional correlations during equity market downturns; namely, when they need it most. The low correlations come from the profits earned on the short leg of the commodity portfolio during commodity market downturns and the partial hedge that these profits provide against the losses on the long leg. As a result, the performance of the LS-IRP commodity portfolio is stable when that of long-only equity indices drops and, consequently, as reported in Panel B of Table 2, the conditional return correlations between the two asset classes do not change in risk-off environments.

Finally, Panel B of Table 2 studies the conditional correlations with fixed income portfolios. Irrespective of the commodity portfolio considered, the β_1 estimates are mostly insignificant, suggesting that the diversification benefits of commodities are the same in calm and turbulent fixed income markets.

4.2. Comparing the risk and performance of traditional and diversified portfolios

Do the added performance of the LO-IRP and LS-IRP commodity portfolios (relative to the S&P-GSCI or the AVG portfolio) and the low correlations between the LS-IRP commodity portfolio and equities convert into better out-of-sample performance for the diversified portfolio? To answer this question, we measure the performance and risks of five portfolios. The first one allocates wealth solely to equities and fixed income securities, the other four portfolios add commodities to the traditional asset mix via either the S&P-GSCI, the AVG commodity portfolio, the LO-IRP commodity portfolio, or the LS-IRP commodity portfolio. As detailed in Section 2.2, we consider various traditional building blocks: i) the S&P-500 index and Barclays bond index in Table 3, ii) the integrated portfolios of stock index and fixed income futures in Table 4 and iii) the characteristic-sorted equity portfolios of Carhart (1997) and Fama and French (2015) and the Barclays bond index in Table 5. In each of the three tables, the asset allocation to the building blocks is based on equal weights in Panel A, optimized weights in Panel B, and volatility-timed weights in Panel C.

Table 3: Risk and performance of traditional (S&P-500 and Barclays bond index) and diversified portfolios

The table reports summary statistics for traditional portfolios made of equities (S&P-500 index) and fixed income securities (Barclays bond index), as well as diversified portfolios that add to the traditional asset mix any one of the following four commodity portfolios: the S&P-GSCI, AVG (a long-only equally-weighted and monthly-rebalanced portfolio of commodity futures), LO-IRP (a long-only integrated risk premium portfolio), or LS-IRP (a long-short integrated risk premium portfolio). The allocation to the equity, fixed income, and commodity building blocks is based on equal weights in Panel A, on optimised weights in Panel B , and on volatility-timed weights in Panel C. Mean is the annualized average return in excess of the risk-free rate. Certainty equivalent, volatility, and downside volatility are annualized. VaR is the monthly Cornish-Fisher Value-at-Risk. Sharpe (Sortino) ratios are annualized mean excess return over annualized volatility (annualized downside volatility using 0% as the threshold). Omega ratios are the probability of gains divided by the probability of losses using 0% as the threshold. Bold fonts denote significance at the 5% level or better. t-statistics are in parentheses. p-value stands for the p-value associated with the null hypothesis that the Sharpe ratio of the traditional portfolio equals that of the diversified portfolio. The sample spans the period from October 1998 to December 2018.

	Traditional portfolio	*Diversified portfolios*			
	Equity & Fixed income	S&P GSCI	AVG	LO IRP	LS IRP
Panel A: Equal allocation to building blocks					
Mean	0.0419	0.0352	0.0359	**0.0668**	**0.0595**
	(1.87)	(1.18)	(1.59)	(2.27)	(2.67)
Cert. Equiv.	**0.0336**	0.0198	0.0272	**0.0551**	**0.0544**
	(2.07)	(0.89)	(1.62)	(2.83)	(4.22)
Volatility	0.07	0.10	0.08	0.09	0.06
Downside volatility	0.05	0.08	0.06	0.06	0.04
Skewness	**-0.66**	**-0.86**	**-0.96**	**-0.39**	-0.25
	(-4.18)	(-5.49)	(-6.11)	(-2.45)	(-1.56)
Ex. kurtosis	**1.81**	**3.75**	**4.33**	**2.47**	**0.83**
	(5.77)	(11.92)	(13.78)	(7.86)	(2.65)
99% VaR (Cornish-Fisher)	-0.05	-0.07	-0.05	-0.07	-0.05
% of positive months	0.65	0.63	0.63	0.63	0.66
Max drawdown	0.15	0.24	0.17	0.17	0.11
Sharpe ratio	0.57	0.35	0.48	0.76	1.02
p-value		0.23	0.45	0.24	0.00
Sortino ratio	0.77	0.45	0.59	1.09	1.55
Omega ratio	1.83	1.50	1.74	2.06	2.73
Panel B: Optimized allocation to building blocks					
Mean	0.0449	0.0499	0.0383	**0.0653**	**0.0572**
	(1.73)	(1.59)	(1.48)	(2.00)	(2.21)
Cert. Equiv.	0.0331	0.0338	0.0265	**0.0500**	**0.0485**
	(1.71)	(1.51)	(1.38)	(2.26)	(2.90)
Volatility	0.09	0.10	0.09	0.10	0.08
Downside volatility	0.06	0.09	0.07	0.08	0.06
Skewness	**-0.63**	**-1.41**	**-1.21**	**-0.65**	**-0.52**
	(-4.00)	(-8.95)	(-7.73)	(-4.12)	(-3.29)
Ex. kurtosis	**1.54**	**7.72**	**5.14**	**2.90**	**2.13**
	(4.89)	(24.56)	(16.37)	(9.22)	(6.78)
99% VaR (Cornish-Fisher)	-0.06	-0.07	-0.06	-0.08	-0.06
% of positive months	0.65	0.64	0.63	0.65	0.67
Max drawdown	0.17	0.28	0.21	0.21	0.15
Sharpe ratio	0.51	0.50	0.44	0.66	0.76
p-value		0.91	0.47	0.34	0.05
Sortino ratio	0.69	0.57	0.53	0.86	0.97
Omega ratio	1.71	1.69	1.64	1.88	2.19
Panel C: Volatility timing allocation to building blocks					
Mean	**0.0280**	**0.0286**	**0.0299**	**0.0328**	**0.0381**
	(2.19)	(2.19)	(2.25)	(2.37)	(2.60)
Cert. Equiv.	**0.0263**	**0.0269**	**0.0283**	**0.0312**	**0.0366**
	(3.59)	(3.68)	(3.93)	(4.27)	(5.23)
Volatility	0.03	0.03	0.03	0.03	0.03
Downside volatility	0.02	0.03	0.03	0.03	0.02
Skewness	**-0.62**	**-0.74**	**-0.72**	**-0.65**	-0.28
	(-3.96)	(-4.74)	(-4.59)	(-4.16)	(-1.78)
Ex. kurtosis	**1.81**	**2.19**	**2.54**	**2.20**	**1.91**
	(5.76)	(6.97)	(8.07)	(7.01)	(6.07)
99% VaR (Cornish-Fisher)	-0.02	-0.02	-0.02	-0.03	-0.03
% of positive months	0.69	0.71	0.70	0.72	0.73
Max drawdown	0.06	0.06	0.06	0.06	0.06
Sharpe ratio	0.85	0.87	0.92	1.00	1.21
p-value		0.58	0.33	0.00	0.00
Sortino ratio	1.15	1.12	1.17	1.30	1.69
Omega ratio	2.75	2.81	3.02	3.13	3.82

Table 4: Risk and performance of traditional and diversified portfolios of integrated risk premia

The table reports summary statistics for traditional portfolios made of integrated risk premium (IRP) portfolios of either stock index futures or fixed income and interest rate futures, as well as diversified portfolios that add to the traditional asset mix any one of the following four commodity portfolios: the S&P-GSCI, AVG (a long-only equally-weighted and monthly-rebalanced portfolio of commodity futures), LO-IRP (a long-only integrated risk premium portfolio), or LS-IRP (a long-short integrated risk premium portfolio). The traditional building blocks are IRP portfolios that are either long-only (left-hand side) or long-short (right-hand side). The allocation to the equity, fixed income, and commodity building blocks is based on equal weights in Panel A, on optimised weights in Panel B, and on volatility-timed weights in Panel C. Mean is the annualized average return in excess of the risk-free rate. Certainty equivalent, volatility, and downside volatility are annualized. Sharpe (Sortino) ratios are annualized mean excess return over annualized volatility (annualized downside volatility using 0% as the threshold). Omega ratios are the probability of gains divided by the probability of losses using 0% as the threshold. VaR is the monthly Cornish-Fisher Value-at-Risk. Bold fonts denote significance at the 5% level or better. t-statistics are in parentheses. p-value stands for the p-value associated with the null hypothesis that the Sharpe ratio of the traditional portfolio equals that of the diversified portfolio. The sample spans the period from October 1998 to December 2018.

	Long-only allocation to equity and fixed income IRP portfolios					*Long-short allocation to equity and fixed income IRP portfolios*				
	Traditional	*Diversified*				*Traditional*	*Diversified*			
		S&P GSCI	AVG	LO IRP	LS IRP		S&P GSCI	AVG	LO IRP	LS IRP
Panel A: Equal allocation to Building Blocks										
Mean	**0.0514**	0.0416	0.0423	**0.0731**	**0.0658**	**0.0465**	0.0383	0.0390	**0.0698**	**0.0625**
	(2.27)	(1.42)	(1.95)	(2.46)	(2.76)	(2.01)	(1.31)	(1.74)	(2.31)	(2.67)
Cert. Equiv.	**0.0425**	0.0278	**0.0344**	**0.0621**	**0.0603**	**0.0380**	0.0244	0.0314	**0.0589**	**0.0572**
	(2.46)	(1.30)	(2.14)	(3.24)	(4.41)	(2.29)	(1.14)	(2.00)	(3.12)	(4.33)
Volatility	0.08	0.10	0.07	0.09	0.06	0.07	0.10	0.07	0.08	0.06
Downside volatility	0.04	0.06	0.05	0.05	0.03	0.06	0.07	0.06	0.06	0.05
Skewness	**0.41**	-0.02	-0.15	0.10	**0.44**	**-0.47**	**-0.50**	**-0.64**	**-0.39**	**-0.65**
	(2.61)	(-0.15)	(-0.99)	(0.62)	(2.82)	(-3.01)	(-3.20)	(-4.06)	(-2.50)	(-4.12)
Ex. kurtosis	**2.04**	0.55	**0.64**	**0.67**	**1.18**	**4.59**	**0.65**	**3.69**	**1.81**	**3.04**
	(6.49)	(1.74)	(2.02)	(2.12)	(3.77)	(14.61)	(2.06)	(11.74)	(5.77)	(9.66)
99% VaR (Cornish-Fisher)	-0.07	-0.07	-0.05	-0.07	-0.06	-0.07	-0.06	-0.06	-0.07	-0.05
% of positive months	0.63	0.62	0.61	0.64	0.67	0.62	0.62	0.61	0.65	0.70
Max drawdown	0.17	0.17	0.13	0.15	0.11	0.19	0.16	0.16	0.16	0.14
Sharpe ratio	0.66	0.43	0.58	0.85	1.07	0.62	0.40	0.55	0.82	1.05
p-value		0.24	0.58	0.28	0.00		0.26	0.62	0.25	0.00
Sortino ratio	1.16	0.70	0.92	1.49	2.08	0.76	0.58	0.70	1.11	1.28
Omega ratio	1.97	1.58	1.86	2.17	2.83	2.04	1.53	1.89	2.21	2.86
Panel B: Optimized allocation to building blocks										
Mean	**0.0557**	0.0555	0.0448	**0.0717**	**0.0644**	**0.0538**	0.0562	0.0474	**0.0777**	**0.0733**
	(2.14)	(1.97)	(1.83)	(2.17)	(2.34)	(1.99)	(1.84)	(1.71)	(2.13)	(2.41)
Cert. Equiv.	**0.0435**	**0.0427**	0.0342	**0.0577**	**0.0556**	**0.0416**	0.0415	0.0339	**0.0597**	**0.0616**
	(2.15)	(2.07)	(1.83)	(2.69)	(3.25)	(2.10)	(1.92)	(1.64)	(2.49)	(3.20)
Volatility	0.09	0.09	0.08	0.10	0.08	0.09	0.10	0.09	0.11	0.09
Downside volatility	0.05	0.05	0.05	0.06	0.05	0.07	0.08	0.08	0.09	0.08
Skewness	**0.41**	0.09	-0.05	0.04	0.13	**-0.48**	**-0.79**	**-1.04**	**-0.87**	**-1.31**
	(2.59)	(0.56)	(-0.34)	(0.28)	(0.85)	(-3.08)	(-5.03)	(-6.64)	(-5.51)	(-8.32)
Ex. kurtosis	**2.03**	**0.65**	0.18	0.47	**0.98**	**5.00**	**1.95**	**5.23**	**3.54**	**5.55**
	(6.45)	(2.08)	(0.58)	(1.50)	(3.11)	(15.92)	(6.19)	(16.63)	(11.28)	(17.65)
99% VaR (Cornish-Fisher)	-0.09	-0.07	-0.06	-0.08	-0.07	-0.08	-0.06	-0.07	-0.08	-0.06
% of positive months	0.61	0.62	0.59	0.63	0.67	0.63	0.64	0.64	0.66	0.70
Max drawdown	0.20	0.17	0.16	0.16	0.14	0.23	0.18	0.23	0.23	0.19
Sharpe ratio	0.61	0.60	0.53	0.74	0.84	0.60	0.58	0.51	0.72	0.85
p-value		0.94	0.44	0.42	0.07		0.86	0.35	0.41	0.01
Sortino ratio	1.07	1.04	0.89	1.28	1.36	0.72	0.73	0.56	0.82	0.86
Omega ratio	1.84	1.78	1.72	1.97	2.26	1.97	1.77	1.79	2.04	2.39
Panel C: Volatility timing allocation to building blocks										
Mean	**0.0347**	**0.0363**	**0.0364**	**0.0462**	**0.0543**	0.0124	0.0129	0.0143	0.0156	0.0200
	(2.20)	(2.11)	(2.19)	(2.43)	(2.65)	(1.21)	(1.21)	(1.33)	(1.40)	(1.66)
Cert. Equiv.	**0.0304**	**0.0322**	**0.0328**	**0.0421**	**0.0509**	0.0112	0.0117	**0.0131**	**0.0144**	**0.0188**
	(2.53)	(2.72)	(2.99)	(3.61)	(4.74)	(1.72)	(1.80)	(2.05)	(2.21)	(2.95)
Volatility	0.05	0.05	0.05	0.05	0.05	0.03	0.03	0.03	0.03	0.03
Downside volatility	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02
Skewness	0.02	0.15	0.21	0.09	**0.32**	-0.19	-0.14	-0.03	-0.13	-0.12
	(0.15)	(0.98)	(1.31)	(0.56)	(2.04)	(-1.23)	(-0.92)	(-0.21)	(-0.84)	(-0.75)
Ex. kurtosis	**2.93**	**2.58**	**1.50**	**2.12**	**1.81**	**1.85**	**1.74**	**1.32**	**1.56**	**1.42**
	(9.34)	(8.21)	(4.79)	(6.76)	(5.77)	(5.89)	(5.53)	(4.19)	(4.96)	(4.52)
99% VaR (Cornish-Fisher)	-0.05	-0.05	-0.04	-0.05	-0.05	-0.02	-0.02	-0.02	-0.02	-0.02
% of positive months	0.64	0.64	0.65	0.64	0.65	0.64	0.65	0.62	0.64	0.65
Max drawdown	0.12	0.12	0.10	0.11	0.10	0.06	0.06	0.06	0.06	0.06
Sharpe ratio	0.64	0.68	0.74	0.88	1.12	0.42	0.44	0.50	0.53	0.70
p-value		0.54	0.43	0.00	0.00		0.53	0.22	0.00	0.00
Sortino ratio	0.92	1.02	1.17	1.35	1.97	0.59	0.63	0.77	0.78	1.06
Omega ratio	2.20	2.27	2.39	2.61	3.28	2.28	2.30	2.40	2.46	2.78

Table 5: Risk and performance of traditional (characteristic-sorted equity portfolios and Barclays bond index) and diversified portfolios

The table reports summary statistics for traditional portfolios made of characteristic-sorted equity portfolios and Barclays bond index, as well as diversified portfolios that add to the traditional asset mix any one of the following four commodity portfolios: the S&P-GSCI, AVG (a long-only equally-weighted and monthly-rebalanced portfolio of commodity futures), LO-IRP (a long-only integrated risk premium portfolio), or LS-IRP (a long-short integrated risk premium portfolio). The equity building blocks are characteristic-sorted portfolios based on Fama and French (2015) and Carhart (1997); they are either long-only (left-hand side) or long-short (right-hand side). The allocation to the equity, fixed income, and commodity building blocks is based on equal weights in Panel A, on optimised weights in Panel B, and on volatility-timed weights in Panel C. Mean is the annualized average return in excess of the risk-free rate. Certainty equivalent, volatility, and downside volatility are annualized. VaR is the monthly Cornish-Fisher Value-at-Risk. Sharpe (Sortino) ratios are annualized mean excess return over annualized volatility (annualized downside volatility using 0% as the threshold). Omega ratios are the probability of gains divided by the probability of losses using 0% as the threshold. p-value stands for the p-value associated with the null hypothesis that the Sharpe ratio of the traditional portfolio equals that of the diversified portfolio. Bold fonts denote significance at the 5% level or better. t-statistics are in parentheses. The sample spans the period from October 1998 to December 2018.

	Long-only allocation to Fama French equity portfolio					*Long-short allocation to Fama French equity portfolio*				
	Traditional	*Diversified*				*Traditional*	*Diversified*			
		S&P GSCI	AVG	LO IRP	LS IRP		S&P GSCI	AVG	LO IRP	LS IRP
Panel A: Equal allocation to building blocks										
Mean	**0.0569**	0.0453	0.0459	**0.0768**	**0.0695**	0.0200	0.0207	0.0213	**0.0522**	**0.0449**
	(2.27)	(1.41)	(1.87)	(2.41)	(2.83)	(1.40)	(0.83)	(1.31)	(2.11)	(2.44)
Cert. Equiv	**0.0472**	0.0284	**0.0364**	**0.0642**	**0.0638**	**0.0183**	0.0114	0.0180	**0.0460**	**0.0428**
	(2.68)	(1.22)	(2.08)	(3.17)	(4.65)	(2.41)	(0.65)	(1.72)	(3.16)	(5.00)
Volatility	0.08	0.10	0.08	0.09	0.06	0.03	0.08	0.05	0.07	0.04
Downside volatility	0.06	0.08	0.06	0.06	0.04	0.02	0.05	0.03	0.04	0.02
Skewness	**-0.73**	**-0.89**	**-1.03**	**-0.45**	-0.28	**0.84**	**-0.37**	**-0.34**	**0.62**	**0.67**
	(-4.62)	(-5.68)	(-6.58)	(-2.85)	(-1.81)	(5.33)	(-2.33)	(-2.17)	(3.93)	(4.26)
Ex. kurtosis	**1.80**	**3.55**	**4.24**	**2.33**	**0.97**	**3.73**	**0.88**	**1.81**	**2.75**	**1.06**
	(5.72)	(11.28)	(13.48)	(7.41)	(3.07)	(11.88)	(2.80)	(5.77)	(8.76)	(3.37)
99% VaR (Cornish-Fisher)	-0.05	-0.07	-0.05	-0.07	-0.05	-0.04	-0.05	-0.04	-0.07	-0.04
% of positive months	0.68	0.63	0.64	0.66	0.71	0.68	0.58	0.60	0.63	0.67
Max drawdown	0.15	0.25	0.16	0.18	0.12	0.07	0.16	0.10	0.13	0.07
Sharpe ratio	0.72	0.43	0.58	0.84	1.12	0.58	0.26	0.45	0.80	1.17
p-value		0.09	0.27	0.42	0.00		0.23	0.57	0.42	0.00
Sortino ratio	0.97	0.54	0.72	1.19	1.69	0.94	0.39	0.67	1.48	2.63
Omega ratio	1.99	1.58	1.86	2.17	2.86	2.62	1.43	1.88	2.29	3.68
Panel B: Optimized allocation to building blocks										
Mean	**0.0629**	0.0634	0.0534	**0.0800**	**0.0724**	0.0186	0.0228	0.0190	**0.0513**	**0.0441**
	(2.19)	(1.87)	(1.90)	(2.26)	(2.55)	(1.23)	(1.03)	(1.12)	(2.12)	(2.45)
Cert. Equiv	**0.0490**	0.0444	0.0396	**0.0625**	**0.0621**	0.0165	0.0168	0.0156	**0.0449**	**0.0419**
	(2.33)	(1.84)	(1.91)	(2.64)	(3.37)	(1.96)	(1.19)	(1.45)	(3.07)	(4.75)
Volatility	0.09	0.11	0.09	0.11	0.08	0.04	0.06	0.05	0.07	0.04
Downside volatility	0.07	0.09	0.08	0.08	0.06	0.02	0.05	0.03	0.03	0.02
Skewness	**-0.70**	**-1.39**	**-1.42**	**-0.78**	**-0.56**	**0.88**	**-0.41**	-0.12	**0.66**	**0.58**
	(-4.44)	(-8.86)	(-9.05)	(-4.98)	(-3.56)	(5.63)	(-2.60)	(-0.79)	(4.18)	(3.72)
Ex. kurtosis	**1.48**	**7.60**	**6.00**	**3.58**	**2.17**	**4.65**	**2.83**	**1.02**	**2.51**	**1.19**
	(4.72)	(24.19)	(19.09)	(11.38)	(6.90)	(14.79)	(9.00)	(3.25)	(7.99)	(3.77)
99% VaR (Cornish-Fisher)	-0.06	-0.08	-0.06	-0.08	-0.06	-0.04	-0.05	-0.04	-0.07	-0.04
% of positive months	0.66	0.65	0.64	0.66	0.67	0.64	0.61	0.58	0.63	0.67
Max drawdown	0.18	0.30	0.22	0.24	0.18	0.08	0.15	0.09	0.13	0.07
Sharpe ratio	0.66	0.58	0.57	0.75	0.88	0.49	0.36	0.39	0.78	1.11
p-value		0.51	0.32	0.53	0.07		0.58	0.63	0.29	0.00
Sortino ratio	0.90	0.68	0.68	0.99	1.15	0.76	0.46	0.60	1.52	2.29
Omega ratio	1.86	1.78	1.77	1.99	2.31	2.36	1.69	1.81	2.24	3.48
Panel C: Volatility timing allocation to Building Blocks										
Mean	**0.0293**	**0.0299**	**0.0311**	**0.0341**	**0.0394**	0.0185	0.0192	0.0206	0.0222	0.0260
	(2.24)	(2.24)	(2.29)	(2.41)	(2.62)	(1.42)	(1.46)	(1.58)	(1.63)	(1.86)
Cert. Equiv	**0.0277**	**0.0283**	**0.0296**	**0.0325**	**0.0379**	**0.0172**	**0.0179**	**0.0194**	**0.0209**	**0.0248**
	(3.82)	(3.91)	(4.14)	(4.50)	(5.45)	(2.55)	(2.68)	(3.07)	(3.18)	(3.90)
Volatility	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Downside volatility	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Skewness	**-0.53**	**-0.66**	**-0.64**	**-0.55**	-0.16	-0.02	0.03	0.06	0.01	0.08
	(-3.40)	(-4.18)	(-4.07)	(-3.53)	(-0.99)	(-0.14)	(0.16)	(0.35)	(0.06)	(0.49)
Ex. kurtosis	**1.77**	**2.08**	**2.34**	**2.10**	**1.90**	**1.72**	**1.71**	**1.43**	**1.53**	**1.20**
	(5.63)	(6.63)	(7.44)	(6.68)	(6.05)	(5.48)	(5.44)	(4.55)	(4.86)	(3.81)
99% VaR (Cornish-Fisher)	-0.03	-0.02	-0.02	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
% of positive months	0.70	0.71	0.71	0.72	0.73	0.66	0.65	0.67	0.68	0.70
Max drawdown	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Sharpe ratio	0.90	0.92	0.97	1.05	1.26	0.61	0.64	0.72	0.75	0.91
p-value		0.60	0.37	0.00	0.00		0.30	0.08	0.00	0.00
Sortino ratio	1.25	1.21	1.25	1.39	1.81	0.93	0.99	1.14	1.17	1.49
Omega ratio	2.87	2.94	3.15	3.28	4.02	2.54	2.62	2.87	2.86	3.24

Bringing together the evidence reported in these three tables, it appears that over the period considered (from October 1998 to December 2018) investors can benefit from adding well-designed commodity portfolios to their strategic mix. Across tables and panels, the Sharpe ratio of traditional portfolios made of stocks and bonds drops on average by 15.3% when adding the S&P-GSCI to the strategic mix and by 5.9% when adding the AVG commodity portfolio. In contrast, the average Sharpe ratio rises by 27.6% when adding the LO-IRP commodity portfolio to the traditional asset mix and by 61.9% when adding the LS-IRP commodity portfolio. The Sortino and Omega ratios provide a similar picture across the panels and tables. For example, the Sortino ratio of the traditional asset mix drops on average by 18.1% and 8.8% when adding the S&P-GSCI and AVG to traditional stock-bond portfolios and rises by 32.9% and 77.7% when adding instead the LO-IRP and LS-IRP commodity portfolios, respectively.[11] The highest Sharpe and Omega ratios (1.26 and 4.02, respectively) are obtained when adding the LS-IRP commodity portfolio to a traditional asset mix made of long-only FF portfolios and Barclays bond index (Table 5, Panel C). The corresponding ratios for the traditional portfolios that exclude LS-IRP are at 0.9 and 2.87, respectively.

The conclusion that adding the LS-IRP commodity portfolio to the traditional asset mix enhances the risk-adjusted performance of stock-bond portfolios holds per panel and per table, suggesting that the inference is not driven by the choice of traditional asset mix (across tables) or by the choice of allocation technique (across panels). The rise in risk-adjusted performance is also often statistically significant: The p-values associated with the null hypothesis that the Sharpe ratio of the asset mix with LS-IRP equals the Sharpe ratio of the asset mix without LS-IRP are often less than 5%, indicating that the rise in Sharpe

[11] Likewise, the Omega ratio of the traditional asset mix drops on average by 11% and 3.6%, when adding the S&P-GSCI and AVG to traditional stock-bond portfolios and rises by 7.8% and 35.9% when adding instead the LO-IRP and LS-IRP commodity portfolios, respectively.

ratio obtained when adding LS-IRP is statistically significant. The evidence is less strong for the LO-IRP commodity portfolio, for which the change in Sharpe ratios obtained when adding LO-IRP to stock-bond portfolios is less often statistically significant.

The substantial increases in risk-adjusted performance observed when adding the LS-IRP commodity portfolio to traditional portfolios made of stocks and bonds are jointly driven by increases in mean excess returns and decreases in risk levels. For example, adding commodities to the strategic mix via LO-IRP and LS-IRP, respectively, increases the mean excess return of the traditional asset mix by 52.1% and 47.4% on average across tables and panels. Looking now at the risk profiles, the asset mix with the LO-IRP or LS-IRP commodity portfolio is found to present on average (i.e., across tables and panels) a level of excess kurtosis that is 1.6% or 23.8% lower, respectively, than the level of excess kurtosis observed for the traditional asset mix. In sharp contract, adding commodities to the traditional asset mix via the S&P-GSCI or AVG raises the excess kurtosis of the traditional asset mix by 42.4% and 32.5%, respectively. Likewise, adding to the traditional stock-bond mix the LS-IRP commodity portfolio lowers on average the maximum drawdown of the traditional stock-bond mix by 13.1%. In contrast, adding the S&P-GSCI or AVG to the traditional stock-bond mix increases the maximum drawdown of the traditional stock-bond mix by 28.1% or 3%, respectively. These results serve to highlight the benefits in terms of risk profile of treating the LS-IRP commodity portfolio as part of the strategic mix of investors.

Comparing the risk-adjusted performance of the diversified portfolios across asset allocation strategies (i.e., across the panels), we note that the superiority of the strategic mix that allocates wealth to the commodity-based LS-IRP portfolio holds, irrespective of the portfolio allocation methods. Nonetheless, investors seem to be better off allocating wealth to the various building blocks using equal or volatility-timed weights (Panels A and C). The relatively worse performance of the diversified strategy based on optimized weights (Panel B) is somehow expected: Optimized weights are known to yield large estimation errors and poor

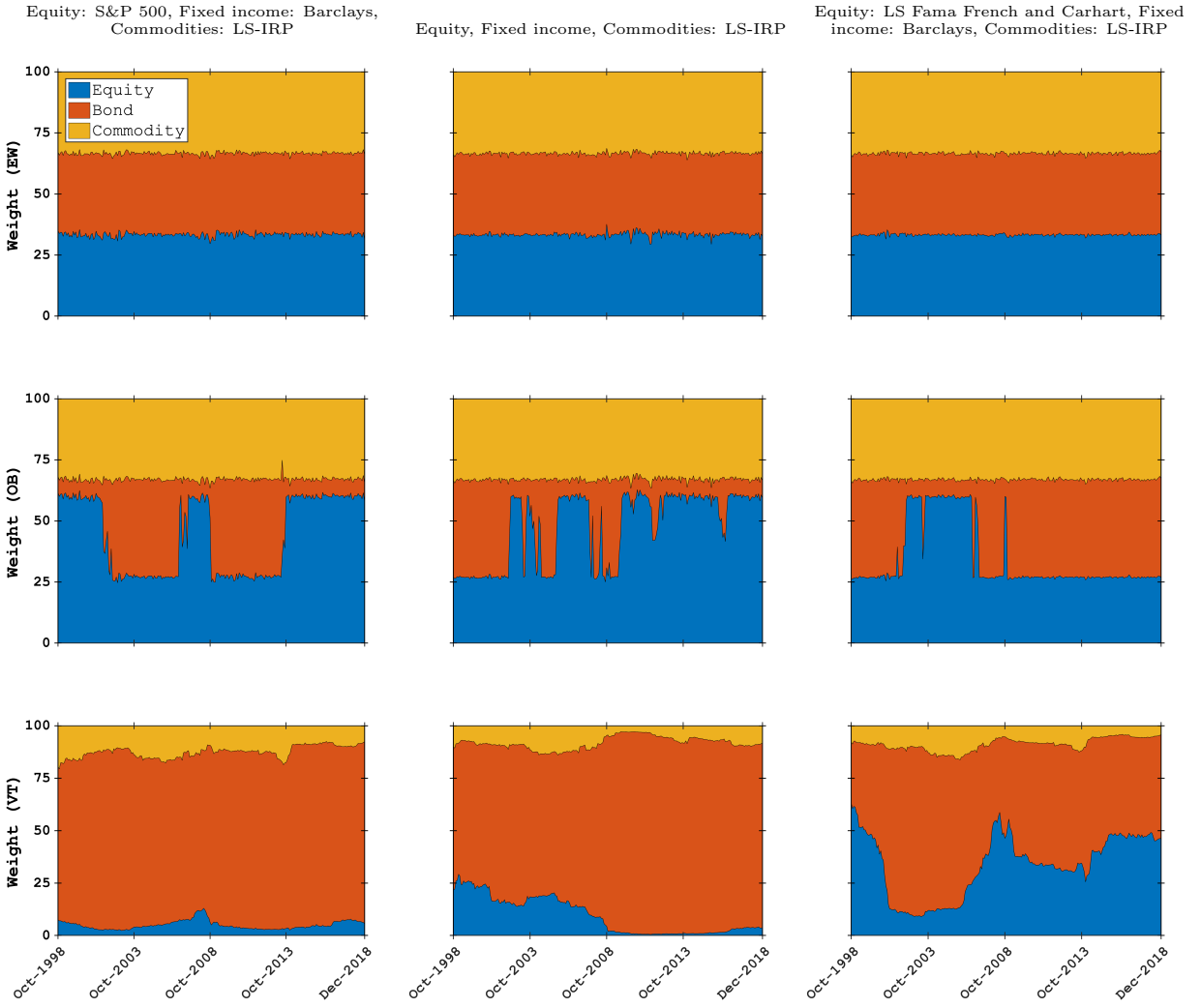
out-of-sample performance (Jagannathan and Ma, 2003; Michaud, 2008; Kirby and Ostdiek, 2012). Imposing upper-bound constraints, as we do, helps to alleviate these issues but does not eliminate them.

4.3. Allocations over time to the various building blocks

Figure 1 shows the allocation over time to the equity, fixed income, and commodity building blocks. The rows pertain to the portfolio allocation methods: equal weights (EW), optimization-based weights (OB), and volatility-timed weights (VT). The columns allow for different traditional building blocks ranging from the S&P-500 index and Barclays bond index (left column) to equity and fixed income LS-IRP portfolios (middle column) and to long-short characteristic-sorted equity portfolios and Barclays bond index (right column). In all settings, the LS-IRP portfolio is used as the commodity building block. This choice is governed by our previous results that have shown the superior performance of the diversified portfolios that make use of LS-IRP.

As expected, the optimization-based (OB) construction method exhibits greater fluctuations in weights across the building blocks, consistent with Black and Litterman (1992) and Best and Grauer (1991). The practical implementation of such an allocation strategy is more challenging due to higher transaction costs and implementation constraints. VT, by definition, constructs allocations that are inversely proportional to the risks of the building blocks. Consequently, fixed income, with inherently lower volatility, constitutes a larger portion of the portfolio. The VT approach responds to changes in the underlying data in an orderly and gradual fashion as opposed to the OB method, whereby weight changes are more sudden and extreme. This is a highly desirable attribute because it allows for practical implementation.

Figure 1: Allocation over time to different asset classes
The figure illustrates the monthly evolution in the weights allocated to the equity, fixed income, and commodity building blocks, whereby the constituents of the building blocks are mentioned in the headings of each column. The allocation to the building blocks is based on equal weights (EW) in Panel A, optimised weights (OB, optimisation-based allocation) in Panel B, and volatility timing (VT) in Panel C. The sample spans the period from October 1998 to December 2018.



4.4. Abnormal performance

Another way to test if commodities add value to a traditional portfolio is to carry out the regression analysis described in Section 2.4.1. Table 6 reports coefficient estimates and Newey and West (1987) adjusted t-statistics from regressions of the excess returns of a diversified portfolio onto the excess returns of its corresponding traditional portfolio (Equation (9)). The results are presented for the different equity, fixed income, and commodity building blocks considered in our paper (as reported in the various rows of the table) and after allowing for various allocation methods to these building blocks (equal, optimized, and volatility-timed weights as reported in Panels A, B, and C, respectively).

When LS-IRP is used as commodity building block, the annualized alphas of the regressions are positive and statistically significant at the 5% level or better, averaging 2.42% a year with a range from 0.84% (t-statistic of 4.46) to 3.36% (t-statistic of 4.92) across portfolios. When using LO-IRP in place of LS-IRP as a commodity building block, the annualized alphas equal 2.28% on average and are statistically significant at the 10% level or better for 11 of the 15 portfolios considered. Confirming previous studies, using naive long-only commodity portfolios (S&P-GSCI and AVG) as building blocks does not add value to investors: The alphas are zero economically (average of 0.27% a year) and statistically, suggesting an absence of diversification benefits. This analysis serves to confirm the superiority of diversified portfolios that allocate wealth to long-only and long-short integrated commodity portfolios.

Table 6: Abnormal performance

The table reports estimated coefficients from regressions of the excess returns of a diversified portfolio onto the excess returns of the corresponding traditional portfolio. Traditional portfolios are constructed using the equity and fixed income building blocks of Tables 3 to 5 as reported in Column 1. Diversified portfolios add to the traditional asset mix any one of the following four commodity portfolios: the S&P-GSCI, AVG (a long-only equally-weighted and monthly-rebalanced portfolio of commodity futures), LO-IRP (a long-only integrated risk premium portfolio), or LS-IRP (a long-short integrated risk premium portfolio). The intercepts have been annualized and the t-statistics in parentheses are Newey and West (1987) adjusted. Adj. R^2 stands for the adjusted goodness-of-fit statistic of the regression. The allocation to the building blocks is based on equal weights in Panel A, optimised weights in Panel B, and volatility timing in Panel C. Bold fonts denote significance at the 5% level or better. The sample spans the period from October 1998 to December 2018.

Traditional	Commodity	*Panel A: Equal weights*					*Panel B: Optimized weights*					*Panel C: Volatility timing weights*				
building blocks	building blocks	α		β		Adj. R^2	α		β		Adj. R^2	α		β		Adj. R^2
S&P 500 and Barclays	S&P GSCI	-0.0035	(-0.16)	**0.92**	(8.73)	0.47	0.0079	(0.45)	**0.94**	(9.64)	0.66	0.0012	(0.73)	**0.98**	(94.28)	0.97
	AVG	-0.0006	(-0.05)	**0.87**	(14.89)	0.72	-0.0020	(-0.26)	**0.90**	(18.26)	0.82	0.0039	(1.44)	**0.93**	(28.76)	0.89
	LO IRP	0.0303	(1.85)	**0.87**	(12.53)	0.53	0.0255	(1.43)	**0.89**	(10.49)	0.60	**0.0057**	(2.71)	**0.97**	(52.95)	0.95
	LS IRP	**0.0313**	(4.70)	**0.67**	(29.03)	0.73	**0.0244**	(2.47)	**0.73**	(10.11)	0.72	**0.0137**	(4.55)	**0.87**	(37.15)	0.85
Long-only IRP equity and fixed income	S&P GSCI	0.0019	(0.10)	**0.77**	(7.59)	0.39	0.0128	(1.01)	**0.77**	(7.14)	0.56	0.0038	(0.98)	**0.94**	(52.32)	0.91
	AVG	0.0033	(0.39)	**0.76**	(12.41)	0.66	-0.0014	(-0.26)	**0.83**	(9.54)	0.81	0.0091	(1.61)	**0.79**	(24.37)	0.74
	LO IRP	**0.0328**	(2.18)	**0.78**	(12.08)	0.50	0.0274	(1.60)	**0.79**	(8.15)	0.56	**0.0147**	(2.93)	**0.91**	(45.56)	0.87
	LS IRP	**0.0306**	(4.72)	**0.68**	(31.49)	0.76	**0.0243**	(2.31)	**0.72**	(10.35)	0.73	**0.0287**	(4.56)	**0.73**	(19.51)	0.69
Long-short IRP equity and fixed income	S&P GSCI	0.0011	(0.06)	**0.80**	(7.48)	0.39	0.0104	(0.75)	**0.85**	(7.40)	0.61	0.0006	(0.66)	**0.99**	(117.64)	0.98
	AVG	0.0036	(0.43)	**0.76**	(11.81)	0.65	-0.0031	(-0.41)	**0.94**	(9.45)	0.81	0.0025	(1.53)	**0.96**	(48.95)	0.93
	LO IRP	**0.0331**	(2.25)	**0.79**	(12.49)	0.48	0.0262	(1.59)	**0.96**	(8.79)	0.63	**0.0034**	(2.88)	**0.99**	(85.09)	0.97
	LS IRP	**0.0308**	(4.65)	**0.68**	(24.83)	0.74	**0.0258**	(2.94)	**0.88**	(13.26)	0.83	**0.0084**	(4.46)	**0.94**	(56.41)	0.92
Long-only FF equity and Barclays	S&P GSCI	-0.0087	(-0.40)	**0.95**	(11.59)	0.52	0.0015	(0.10)	**0.98**	(15.56)	0.74	0.0012	(0.74)	**0.98**	(99.71)	0.97
	AVG	-0.0025	(-0.25)	**0.85**	(17.21)	0.74	-0.0034	(-0.44)	**0.90**	(21.28)	0.84	0.0039	(1.45)	**0.93**	(29.17)	0.89
	LO IRP	0.0276	(1.72)	**0.86**	(15.40)	0.57	0.0235	(1.44)	**0.90**	(12.95)	0.64	**0.0057**	(2.72)	**0.97**	(54.92)	0.95
	LS IRP	**0.0311**	(4.68)	**0.67**	(30.00)	0.76	**0.0255**	(2.90)	**0.75**	(11.01)	0.74	**0.0138**	(4.56)	**0.87**	(36.99)	0.85
Long-short FF equity and Barclays	S&P GSCI	0.0081	(0.42)	**0.63**	(5.02)	0.07	0.0092	(0.57)	**0.73**	(6.46)	0.19	0.0010	(1.10)	**0.98**	(131.31)	0.98
	AVG	0.0096	(0.95)	**0.59**	(11.41)	0.18	0.0051	(0.52)	**0.75**	(8.86)	0.34	**0.0039**	(2.08)	**0.90**	(38.27)	0.92
	LO IRP	**0.0399**	(2.68)	**0.61**	(5.68)	0.10	**0.0417**	(2.86)	**0.51**	(4.06)	0.08	**0.0044**	(3.20)	**0.96**	(80.06)	0.97
	LS IRP	**0.0315**	(4.91)	**0.67**	(12.64)	0.36	**0.0336**	(4.92)	**0.57**	(6.48)	0.30	**0.0095**	(4.52)	**0.89**	(50.47)	0.91

4.5. Alternative style-integration approach

The asset pricing literature has identified two ways to combine styles into a unique portfolio. The first approach is the one-step strategy discussed thus far. The second is based on two steps that consist of first forming K single-style portfolios and then constructing another portfolio that allocates equal wealth to these K portfolios. Asset managers who already manage K single-style portfolios could offer this integrated portfolio to their clients and see it as a viable alternative to the one-step solution discussed thus far. It remains to be tested whether the diversification benefits identified with the one-step integration approach persist in the context of this two-step solution.[12]

Table 7 reports the results. The first set of columns presents summary statistics for the commodity-based integrated portfolio based on the two-step approach. The results are presented in both a long-only (LO) and a long-short (LS) settings. Comparing these summary statistics to those reported in Table 1, it appears that the one-step integrated portfolios perform slightly better on a risk-adjusted basis than the corresponding two-step integrated portfolios. For example, the Sharpe ratio of the LO-IRP portfolio is 0.63 based on the one-step approach (Table 1) versus 0.43 based on the two-step approach (Table 7). The corresponding figures for the LS-IRP portfolios are 1.02 for the one-step approach (Table 1) versus 0.97 for the two-step approach (Table 7).

Next, we study the diversification benefits obtained when adding the two-step integrated portfolio of commodities to the traditional building blocks. As in Tables 3 to 5, we consider various sets of traditional building blocks such as i) the S&P-500 and Barclays bond index, ii) long-only or long-short integrated portfolios of equity and fixed income futures, and iii) long-only or long-short characteristic-sorted equity portfolios and Barclays bond index.

[12]The literature on the relative merits of the two integration approaches is inconclusive: Some studies endorse the one-step integrated portfolio (Bender and Wang, 2016; Fitzgibbons et al., 2017; Clarke et al., 2018), while other studies suggest that neither of the methods stands out (Fraser-Jenkins et al., 2016; Leippold and Rueegg, 2018).

We then add to these traditional portfolios either the LO or the LS two-step integrated portfolio of commodities, allowing for various allocations to the different building blocks (equal weights in Panel A, optimized weights in Panel B, and volatility-timed weights in Panel C). Finally, we compare the performance of the then-obtained diversified portfolios to that of the previously-reported traditional and diversified portfolios (Tables 3 to 5).

We draw two conclusions. First, investors should consider the two-step integrated portfolio of commodities, rather than the S&P-GSCI or the AVG portfolio, as part of their strategic mix, as the risk-adjusted performance is then higher. This conclusion holds for both the LO and LS two-step integrated portfolios, irrespective of the traditional building blocks considered or the weighting scheme adopted. All in all, these results serve to highlight once more the diversification benefits of style integration. Second, the choice between the one-step or the two-step integration method depends on the allocation chosen for the various building blocks. Investors who opt for the equal (Panel A) or the optimized (Panel B) weighting scheme should select the one-step integrated portfolio (rather than its two-step counterpart), as the risk-adjusted performance of the diversified portfolios is then higher. However, investors interested in volatility timing (Panel C) should allocate wealth to the two-step integrated portfolio of commodities, as the then-resulting diversified portfolios present higher risk-adjusted returns. To understand this extra performance, it is important to note the low volatility of the two-step integrated portfolio (at 5% in Table 7) relative to its one-step counterpart (at 9% in Table 1) and to recall that the volatility-timing scheme assigns (by construction) higher weights to low-volatility portfolios. As a result, the portfolios in Panel C, Table 7 present higher performance as they allocate relatively more weights to the integrated portfolio of commodities than the portfolios in Panel C of Tables 3 to 5.

Table 7: Risk and performance of portfolios using two-step integration approach
The table reports summary statistics for the two-step integrated portfolio of commodities and for the corresponding diversified portfolios. The diversified portfolios include this two-step integrated commodity portfolio as well as the following traditional asset classes: the S&P-500 index and Barclays bond index (under label (1)), long-only or long-short futures-based equity and fixed income integrated portfolios (under label (2)) and long-only or long-short characteristic-sorted portfolios of equities and Barclays bond index (under label (3)). The allocation to the equity, fixed income, and commodity building blocks is based on equal weights in Panel A, optimised weights in Panel B, and volatility-timed weights in Panel C. Mean is the annualized average return in excess of the risk-free rate. Volatility and downside volatility are annualized. VaR is the monthly Cornish-Fisher Value-at-Risk. Sharpe (Sortino) ratios are annualized mean excess return over annualized volatility (annualized downside volatility using 0% as the threshold). Omega ratios are the probability of gains divided by the probability of losses using 0% as the threshold. Bold fonts denote significance at the 5% level or better. t-statistics are in parentheses. p-value stands for the p-value associated with the null hypothesis that the Sharpe ratio of the traditional portfolio equals that of the diversified portfolio. The sample spans the period from October 1998 to December 2018.

	Two-step commodity portfolio		*Diversified portfolios*					
			(1)		*(2)*		*(3)*	
	LO	LS	LO	LS	LO	LS	LO	LS
Panel A: Equal allocation to building blocks								
Mean	0.0635	**0.0473**	0.0483	**0.0440**	**0.0546**	**0.0470**	**0.0583**	**0.0294**
	(1.74)	(3.62)	(1.87)	(2.37)	(2.13)	(2.40)	(2.08)	(2.03)
Volatility	0.15	0.05	0.08	0.05	0.08	0.05	0.08	0.03
Downside volatility	0.10	0.03	0.06	0.04	0.05	0.04	0.06	0.01
Skewness	0.05	0.65	-0.75	-0.53	-0.05	-0.66	-0.80	0.67
	0.41	**5.77**	**-4.78**	**-3.36**	-0.34	**-4.23**	**-5.07**	**4.28**
Ex. kurtosis	(2.20)	(1.73)	(3.31)	(1.55)	(0.50)	(3.96)	(3.20)	(1.05)
	9.74	**7.66**	**10.53**	**4.94**	1.60	**12.60**	**10.20**	**3.35**
99% VaR (Cornish-Fisher)	(-0.13)	(-0.05)	(-0.06)	(-0.04)	(-0.06)	(-0.05)	(-0.06)	(-0.03)
% of positive months	0.35	0.40	0.63	0.65	0.61	0.70	0.65	0.68
Max drawdown	0.31	0.10	0.17	0.10	0.15	0.13	0.17	0.05
Sharpe ratio	0.43	0.97	0.60	0.85	0.70	0.88	0.70	1.02
p-value			0.82	0.00	0.50	0.26	0.41	0.17
Sortino ratio	0.66	1.87	0.79	1.17	1.16	1.05	0.90	2.33
Omega ratio	1.40	2.15	1.87	2.45	1.99	2.63	1.99	3.69
Panel B: Optimized allocation to building blocks								
Mean	0.0635	**0.0473**	0.0495	0.0404	0.0544	**0.0581**	**0.0632**	0.0271
	(1.74)	(3.62)	(1.74)	(1.83)	(1.92)	(2.12)	(2.05)	(1.96)
Volatility	0.15	0.05	0.09	0.07	0.09	0.08	0.10	0.03
Downside volatility	0.10	0.03	0.07	0.06	0.05	0.08	0.08	0.02
Skewness	0.05	0.65	-1.01	-0.76	0.00	-1.27	-1.17	0.49
	0.41	**5.77**	**-6.40**	**-4.84**	0.03	**-8.05**	**-7.46**	**3.10**
Ex. kurtosis	(2.20)	(1.73)	(4.63)	(2.93)	(0.49)	(6.07)	(5.59)	(1.22)
	9.74	**7.66**	**14.72**	**9.31**	1.56	**19.31**	**17.80**	**3.87**
99% VaR (Cornish-Fisher)	(-0.13)	(-0.05)	(-0.06)	(-0.05)	(-0.07)	(-0.06)	(-0.07)	(-0.03)
% of positive months	0.35	0.40	0.65	0.64	0.63	0.70	0.64	0.65
Max drawdown	0.31	0.10	0.20	0.15	0.16	0.19	0.23	0.06
Sharpe ratio	0.43	0.97	0.56	0.58	0.63	0.70	0.66	0.91
p-value			0.74	0.50	0.52	0.46	0.29	0.24
Sortino ratio	0.66	1.87	0.67	0.70	1.06	0.71	0.80	1.79
Omega ratio	1.40	2.15	1.80	1.92	1.85	2.19	1.91	3.32
Panel C: Volatility timing allocation to building blocks								
Mean	0.0635	**0.0473**	**0.0316**	**0.0354**	**0.0424**	0.0206	**0.0329**	0.0254
	(1.74)	(3.62)	(2.33)	(2.53)	(2.33)	(1.71)	(2.37)	(1.90)
Volatility	0.15	0.05	0.03	0.03	0.05	0.03	0.03	0.03
Downside volatility	0.10	0.03	0.03	0.02	0.03	0.02	0.02	0.01
Skewness	0.05	0.65	-0.70	-0.09	0.13	-0.02	-0.60	0.14
	0.41	**5.77**	**-4.42**	-0.57	0.83	-0.11	**-3.79**	0.91
Ex. kurtosis	(2.20)	(1.73)	(2.35)	(1.75)	(1.88)	(0.91)	(2.22)	(0.77)
tstat	**9.74**	**7.66**	**7.48**	**5.57**	**5.97**	**2.91**	**7.05**	**2.46**
99% VaR (Cornish-Fisher)	(-0.13)	(-0.05)	(-0.02)	(-0.03)	(-0.05)	(-0.02)	(-0.03)	(-0.02)
% of positive months	0.35	0.40	0.72	0.72	0.64	0.65	0.73	0.70
Max drawdown	0.31	0.10	0.06	0.05	0.11	0.06	0.06	0.05
Sharpe ratio	0.43	0.97	0.97	1.24	0.82	0.78	1.02	0.97
p-value			0.03	0.00	0.94	0.88	0.01	0.49
Sortino ratio	0.66	1.87	1.24	1.87	1.30	1.30	1.33	1.75
Omega ratio	1.40	2.15	3.09	4.12	2.51	3.00	3.23	3.53

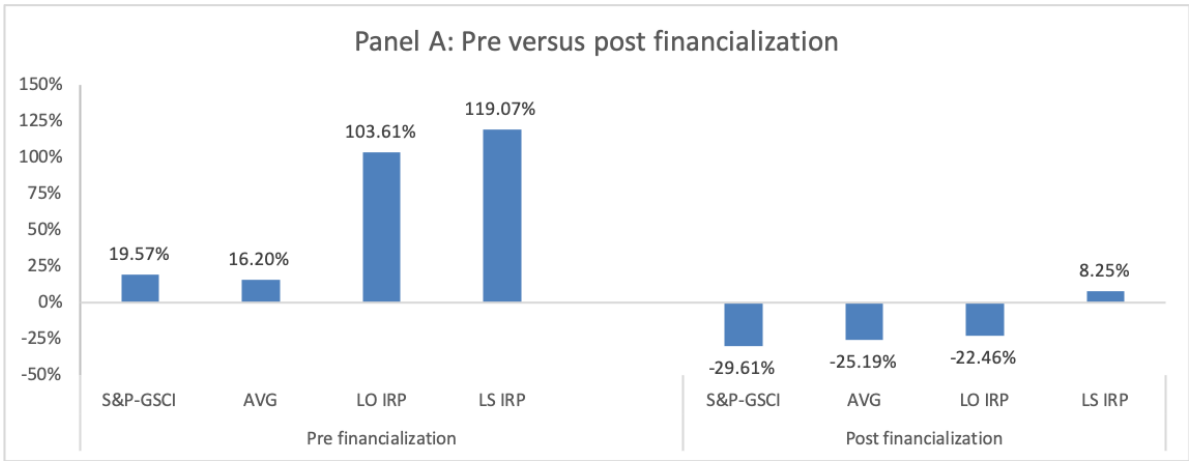
4.6. Are the results sample-specific?

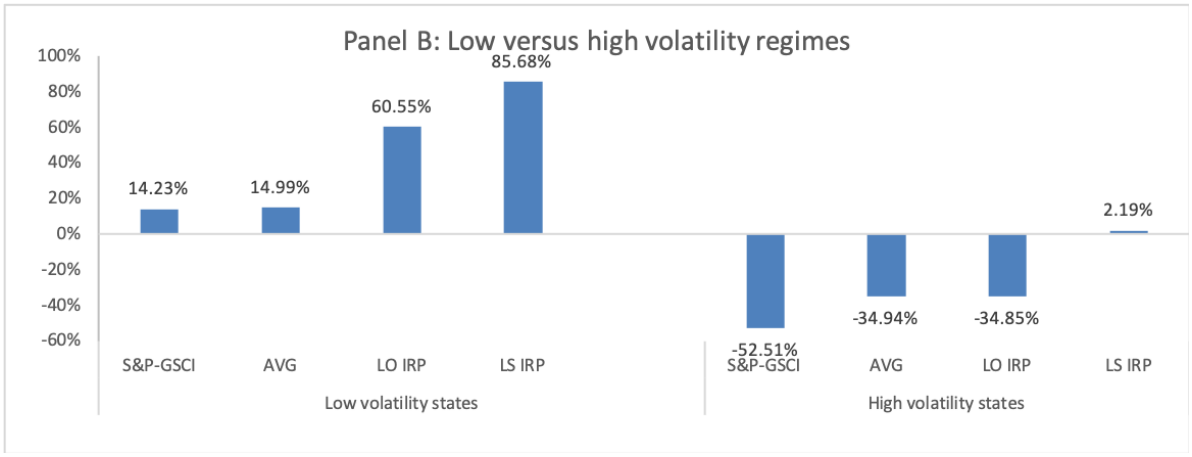
To address this question, we re-evaluate the performance of the traditional and diversified portfolios over different sub-periods such as i) the periods pre and post the financialization of commodity futures dated January 2006 (Stoll and Whaley, 2010), ii) periods of high and low volatility in commodity futures markets,[13] and iii) periods of expansion or recession as defined by the National Bureau of Economic Research. In each of the subperiods, we measure the Sharpe ratios of the traditional portfolios made of the S&P-500 and Barclays bond index and compare them to those of a diversified version thereof that includes any one of the following four commodity portfolios: the S&P-GSCI, the AVG portfolio, the LO-IRP portfolio, or the LS-IRP portfolio.

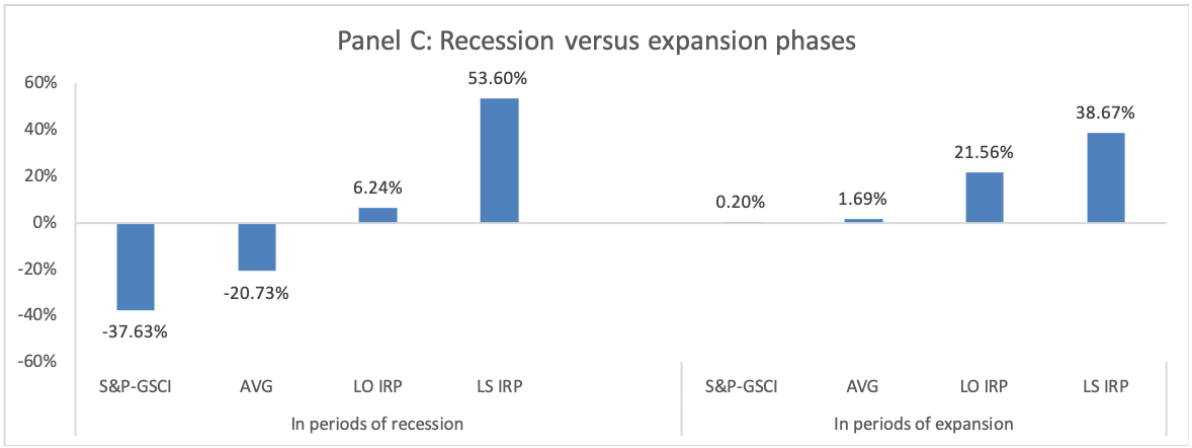
Figure 2 presents the percentage change in Sharpe ratios that is obtained per sub-period when considering commodities as part of the strategic mix. The figure confirms our main findings: Irrespective of the subperiod considered, adding the LO-IRP or the LS-IRP commodity portfolio to the traditional strategic mix improves risk-adjusted performance, while adding the S&P-GSCI or AVG lowers risk-adjusted performance. We note that the benefits of style integration for risk diversification are particularly strong pre-financialization (Panel A), in periods of low volatility in commodity markets (Panel B), and, aligned with the results of Panel B of Table 2, during equity market downturns (Panel C).

[13]High versus low commodity market volatility regimes are identified by fitting a GARCH(1,1) model to the monthly excess returns of the AVG portfolio. The threshold to define the regimes is the average fitted volatility (12.1% a year).

Figure 2: Subperiod analysis
The figure illustrates the diversification benefits of commodities over different subperiods, such as pre and post the financialisation of commodity futures markets (Panel A), high and low volatility regime in commodity markets (Panel B), and phases of recession and expansion (Panel C). The figure presents the percentage change in Sharpe ratios that is obtained per sub-period when adding commodities to a strategic mix made of S&P-500 and Barclays indices.







5. Conclusions

Our article contributes to the literature by intersecting our knowledge of the diversification benefits of commodity futures with the idea of style integration. Instead of considering commodity building blocks that are poorly designed, we contemplated the possibility that investors allocate their wealth to the long-only or long-short integrated portfolio of Fernandez-Perez et al. (2019), whereby these portfolios are designed to combine the signals that matter to the pricing of commodity futures (in our setting, carry, momentum, value, skewness, liquidity, hedging pressure, basis-momentum, relative basis).

In particular, we view the long-short integrated portfolio as a worthy candidate for inclusion into a traditional portfolio made of stocks and bonds for many reasons. First, with an impressive Sharpe ratio of 1.02, it presented remarkable performance and risk profiles compared to all the other portfolios considered in this study. Second, the return correlations between equities and the long-short integrated commodity portfolio were lower than those obtained between equities and the other commodity portfolios considered in this study. Third, the conditional correlations remained low during periods of increased volatility in traditional asset markets, suggesting that the benefits of diversification are present when they are most needed. This result contrasted sharply with that obtained for naive long-only commodity portfolios, whose correlations with equity returns tended to rise during periods of heightened volatility in equity markets.

Bearing all this in mind, we measured the diversification benefits of the long-short integrated commodity portfolio by comparing the performance and risk profiles of a diversified portfolio that treats it as a building block to the performance and risk profiles of a traditional portfolio that solely invests in equities and fixed income securities. For more generality and robustness, the analysis allowed for long-only and long-short building blocks as well as various allocation techniques for these building blocks (equal, optimized, and volatility-timed

weighting schemes).

The results highlight the diversification benefits obtained when adding the long-short integrated portfolio of commodity futures to the traditional asset mix of investors. Across traditional building blocks and weighting schemes, the Sharpe ratios of the diversified portfolios were on average 61.9% higher than the Sharpe ratios of the corresponding traditional portfolios of stocks and fixed income securities. Regression analysis results demonstrated the presence of positive and statistically significant alphas that averaged 2.42% a year across models. Albeit to a lower extent than with a long-short allocation, style integration also added value in a long-only setting: The average increase in Sharpe ratios compared to the traditional strategic mix then equaled 27.6% across traditional building blocks and weighting schemes. Reassuringly, this result indicates that the benefits of diversification are not solely the appanage of asset managers with long-short mandates. Finally, we showed that the conclusions were robust to the choice of equity building blocks and portfolio allocation methods, and robustly persisted when we considered a two-step approach to style integration as well as various sub-periods.

Accordingly, the practical implication of our work suggests that investors will benefit from incorporating commodity risk premia into their strategic mix, provided they are selective in the type of commodity exposure they choose. Naive exposures such as those provided by the S&P-GSCI index or a naive long-only equally-weighted portfolio of all commodities fail to bring significant diversification benefit to the investment portfolio. To realize the enhanced out-of-sample performance that commodity exposure can bring to investors in a multi-asset investment portfolio, sophisticated approaches based on long-only or long-short style integration should be used.

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Appendix A. GARCH(1,1) and DCC(1,1) methodologies

The generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1986) measures the time t variance, $h_{i,t}$, of a return series, $R_{i,t}$, as

$$R_{i,t} = \mu + \epsilon_{i,t}, \tag{A.1}$$

$$h_{i,t} = \gamma + \alpha\epsilon_{i,t-1}^2 + \beta h_{i,t-1}, \tag{A.2}$$

where $\epsilon_{i,t} \sim N(0, h_{i,t})$, μ is the mean of the return series and α, β and γ are coefficients with the following constraints: $\gamma > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta < 1$.

We use the dynamic conditional correlation (DCC) model of Engle (2002) to estimate the pairwise time-varying correlations between building blocks i and j. DCC is implemented in 2 steps. The first step involves estimating the univariate conditional volatilities, $\sqrt{h_{i,t}}$, using the GARCH(1,1) model. The second step uses the standardized residuals from the first step to estimate the time-varying conditional correlations denoted via the matrix R_t:

$$H_t = D_t R_t D_t, \tag{A.3}$$

where $D_t = diag\{\sqrt{h_{i,t}}\}$. R_t can be estimated by:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{A.4}$$

$$Q_t = (1 - a - b)\bar{Q} + a(\epsilon_{i,t}\epsilon_{j,t}) + bQ_{t-1}, \tag{A.5}$$

where $\epsilon_{i,t} = R_{i,t}/\sqrt{h_{i,t}}$ and $\epsilon_{j,t} = R_{j,t}/\sqrt{h_{j,t}}$ are the standardized residuals of building blocks i and j from the first step, $\bar{Q}$ is the unconditional covariance matrix of the standardized residuals, and a and b are coefficients with the following constraints: $a \geq 0$, $b \geq 0$, and $a + b < 1$. $Q_t^* = diag\{q_{ii,t}\}$ is a diagonal matrix with its elements $q_{ii,t}$ defined as the square root of the ith diagonal element of Q_t. The conditional correlation between building blocks i and j at time t is therefore given by:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}}\sqrt{q_{jj,t}}} \tag{A.6}$$

Appendix B. List of stock index, fixed income and interest rate futures

This table presents the list of stock index futures and fixed income and interest rate futures that are used to construct the long-only and long-short integrated portfolios of traditional assets.

Equities	*Fixed Income*
Dow-Jones Industrial Average	1-Month Eurodollar
E-mini Dow-Jones Industrial Average	30-Day FED Funds
E-Mini S&P500	90-Day U.S. Treasury Bill
Euro Stoxx 50	3-Month CD
Eurotop 100	3-Month Eurodollar
Eurotop 300	3-Month Euromark
Major Market Index	2-Year U.S. Treasury Note
MSCI Asia	3-Year U.S. Treasury Note
MSCI EAFE	5-Year Eurodollar Bundle
MSCI Emerging Markets	5-Year U.S. Treasury Note
MSCI Emerging Markets Latin America	10-Year Agency Note
MSCI India	10-Year U.S. Treasury Note
MSCI Russia	30-Year U.S. Treasury Bond
MSCI Taiwan	BC U.S. Aggregate
MSCI Thailand	Brazil C Barra Index
MSCI USA	Brazil EI Bond Index
MSCI World	GNMA Constant Default Rate
Nasdaq 100	Mexican Brady Bond Index
Nasdaq Biotechnology	Moodys Bond Index
Nikkei 225	Municipal Bond Index
NYSE composite	Ultra 10-Year U.S. Treasury Note
PSE Technology	Ultra Treasury Bond Index
Russell 1000	
Russell 1000 Growth	
Russell 1000 Value	
Russell 2000	
Russell 2000 Growth	
Russell 2000 Value	
Russell 3000	
S&P Citigroup Growth	
S&P Citigroup Value	
S&P Consumer Discretionary	
S&P Consumer Staples	
S&P Energy	
S&P Finance	
S&P Health	
S&P Industrial	
S&P Information Technology	
S&P Materials	
S&P Small Capitalization	
S&P Utilities	
S&P400 Mid Capitalization	
S&P500	
Value Line	
VIX	